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ACCELERATED PROGRAM IN ELEMENTARY-SCHOOL MATHEMATICS—THE FOURTH YEAR[1]

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This report describes the fourth year (1966-67) of a longitudinal study of the accelerated program in elementary-school mathematics conducted by the Institute. The first year of the study, including details of the procedures by which the students were selected, was reported in Suppes and Hansen (1965). The second year of the study was reported in Suppes (1966) and the third year was reported in Suppes and Ihrke (1967). This report was written to be as homogeneous as possible with the earlier ones. Results are presented in formats similar to those used in previous reports. Additional tables and figures reflect the expansion of the curriculum and

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additional data available from the use of computer-assisted instruction. The second section of this report contains a description of the curriculum for the current year. The third and fourth sections describe the group composition and class procedure. The fifth section presents the systematic behavioral data collected.

The 30 students who participated in the 1966-67 program were bright fourth graders in the fourth year of an accelerated program in elementary-school mathematics. This fact should be considered when reading the description of the curriculum.

Curriculum Description

Sets and Numbers

The students continued to use the *Sets and Numbers* textbook series for their basic classroom instruction. Each student proceeded from the last problem he had completed the previous year to as far as he could progress at his individual rate when the fourth school year ended. During the school year students worked in Books 3B, 4A, 4B, 5, and 6 (Books 3B, 4A, and 4B were preliminary editions of Books 3 and 4). Chapter descriptions are included in Table 2. The following topics are presented in the *Sets and Numbers* texts: numbers and numerals, place value, addition, subtraction, multiplication, division, sets, fractions, measurements, geometry, problem-solving, equations and inequalities, graphs and charts, laws of arithmetic, and logic.

The student proceeded through his *Sets and Numbers* work by reading examples when a new concept was introduced and by asking the teacher for any additional instruction he desired. The student was allowed to skip as many as half of the problems on a text page if he felt he understood the material; but, if after checking his work the teacher decided the student's error rate was too great, he was asked to solve additional problems on the page after he had corrected his errors.

Probability

Although the fourth- and fifth-grade *Sets and Numbers'* texts each contain a chapter on probability, it was decided to give this special group of students a more extensive introduction to probability. During the month of February the first author taught nine sessions with each of the two classroom groups.

Session 1 dealt with an intuitive consideration of the possible outcomes that could be expected in flipping a coin. The students were asked to estimate how many heads they would expect in a hundred flips and in a thousand flips. A simple experiment with ten flips was carried out in class. The children were assigned the homework problem of flipping a coin a hundred times and observing how many heads were found.

Session 2 began with a discussion of the results of the homework experiment. The students were encouraged to conceptualize why a certain variability in result was obtained and also to explain why the variability was limited. This session then turned to the problem of representing the set of possible experimental outcomes and how events could be thought of as sets. The particular example dealt with in detail was the set of possible outcomes of flipping a coin twice, the event of at least one head, the event of exactly one tail, the event of a head on the first flip, and the event of at most one head. The students were given the homework assignment of finding the set of possible outcomes for three flips of a coin and describing the following events as subsets of this set of possible outcomes: the event of at least one head, the event of exactly two tails, the event of at least one head and at least one tail, and the event of at most two heads.

Session 3 discussed the homework from Session 2 and then also turned to the discussion of the chapter on probability in *Sets and Numbers*, Book 5 with an emphasis on the various types of spinners discussed in that chapter. These simple spinners, familiar in children's games, were used throughout as an alternative to the flipping of coins and the distribution of sex of children in a family. Conceptually an emphasis was put on distinguishing between a symmetrical and a nonsymmetrical spinner. There was considerable discussion of how one could estimate probabilities in a nonsymmetrical spinner. The session also dealt with the problem of representing the set of possible outcomes from spinning a two-color or three-color spinner twice.

Session 4 continued the discussion of some of the exercises in *Sets and Numbers*, Book 5 on probability and then turned to an extensive discussion of the distribution of boys and girls in families of three children. As before, the first problem was to work out explicitly the set of possible outcomes, then to deal with various events and their probabilities, for example, the probability of the event of at least two boys and the probability of the event of exactly one girl.

Session 5 dealt once again with the flipping of a coin three times but with an emphasis on determining the probability of a number of different events and a first introduction to the union of two events and the probability of the union when the events are disjoint.

No emphasis was placed in the initial discussion on the necessity of mutual exclusiveness holding in order for the law of addition to hold, but at the end of the session an example was given to show that the addition of probabilities will not always work, and a discussion was begun of the conditions under which additivity would hold. The session ended with a simple statement of the ordinary law of addition for exclusive events.

Session 6 concentrated on determining the number of possible outcomes when a coin is flipped 10 times. The objective was to come up with the standard formulation that with n flips there are 2^n possible outcomes. A similar discussion and a similar final result was obtained in the case of a three-color spinner with of course the base 2 being replaced by base 3.

Session 7 continued the discussion of the number of possible outcomes. Spinners with various numbers of colors were considered. The question was posed of how many flips of a coin would be needed to have about 1,000 possible outcomes. The discussion went on to consider the number of possible outcomes for different numbers of flips, running up to 20, and there was a discussion of how we could estimate approximately the number of possible outcomes for number of flips above 10.

Session 8 returned to the example of three flips of a coin and consideration of the additivity of events. The law of addition was approached in a slightly different way, but the same systematic statement was obtained as in the case of the earlier session on this same topic.

Session 9 reviewed the various topics discussed in the previous sessions and once again stressed the conditions under which the law of addition holds, with the consideration of counterexamples when the two events being considered have a nonempty intersection.

Logic

The logic program initiated during the summer session of 1965 and continued during the 1965-66 school year was expanded during the 1966-67 school year. Since the teletype machines connected to the PDP-1 computer at Stanford University were to be used by these students for an arithmetic drill-and-practice program, an earlier plan of presenting logic via computer-assisted instruction was revived. Presenting the logic curriculum in a programmed format allowed students to proceed at an individual rate through the linear structure of the program. Lessons were prepared for two courses of study, sentential logic and elementary algebra. Both courses used the same logic program, but had separate introductory tracks for rule names and applications. For most of the year the sentential logic stressed derivations using symbols, and the algebra emphasized numerical derivations; however, rules from both were required for some proofs near the end of the year. Each child alternated his course of study from one day to the next; logic one day, algebra the next.

The fourth year's logic program was intended to be self-contained as tutorial computer-assisted instruction at a teletype terminal, but students were able to question a staff member who was available in the teletype room when the logic program was running. Although a considerable amount of individual instruction was given to some students while they were working at the terminals, very little group instruction occurred.

The format used for the logic problems was similar to that used for computer-assisted instruction during the summer of 1965 and for the classroom materials used during the school year 1965-66.

Lesson 1 of the sentential logic contained 19 problems that were written in symbolic format with two or three premises and that required one-step proofs applying *modus ponendo ponens*, a rule of inference familiar to all the students. The rule was abbreviated AA for Affirm the Antecedent. The students needed to know that 'R → S' meant 'if R then S', that 'R → S' was a conditional sentence whose antecedent was R and consequent was S, that 'P' was the abbreviation for 'premise', and that the use of AA required two line numbers with the line number of the conditional sentence followed by the line number of its antecedent. A period separated the two line numbers. After the teletype had printed what the student was to derive and the given premises, the typewheel positioned itself for the student's instructions. The student then typed the abbreviation for the rule and the line numbers required for its application. The next information printed by the teletype was either a valid step based on the student's input or an error message if the student had given instructions for an invalid step. The teletype proceeded to the next problem when the student had completed the desired derivation. An example from Lesson 1 is the following:

Derive:		L
P	(1)	K → L
P	(2)	M
P	(3)	K
AA1.3	(4)	L.

The underlined phrase indicates what the student typed for this problem. The remainder of the typing was performed automatically under computer control.

Lesson 2 contained 8 more problems that had either two or three premises and that required only a one-step proof. Mathematical sentences were included, as well as the usual symbols of sentential logic. Each of the 7 problems in Lesson 3 had three premises and used *modus ponendo ponens*. Two-step problems were presented for the first time in this lesson.

The Rule of Conjunction was introduced in Lesson 4 as the rule that would Form a Conjunction (FC). The 17 problems in this lesson involved one-step, two-step, and three-step derivations using *modus ponendo ponens* and the Rule of Conjunction.

In Lesson 5 the Rule of Simplification was presented as two separate commands for the student to give the computer: to derive the Left Conjunct he typed LC, or to derive the Right Conjunct he typed RC with a designated line number to complete the instruction. For example:

Derive:		R
P	(1)	S → R & Q
P	(2)	S
AA1.2	(3)	R & Q
LC3	(4)	R.

The underlined sections of the problem indicate the student's input for the derivation. There were 21 problems in Lesson 5 that involved one-step, two-step, and three-step derivations that used from one to five premises.

In Lesson 6 there were 20 problems that contained two, three, or four premises using all the rules introduced up to that point in the curriculum. The problems required from one-step to four-step derivations. Another new rule, *modus tollendo ponens*, was introduced as the rule that would Deny a Disjunct (DD). For example:

Derive:		D
P	(1)	A v (B & C)
P	(2)	D v −B
P	(3)	−A
DD1.3	(4)	B & C
LC4	(5)	B
DD2.5	(6)	D.

As before, the underlined sections indicate the student's typed work, and the teletype printed the remainder of the problem.

Lesson 7 contained 21 problems that required from one to four lines to solve problems based on two or three premises. Another new rule, *modus tollendo tollens*, was introduced as Deny the Consequent (DC). The underlined statements represent the student's work in the following problem:

Derive:		R
P	(1)	N
P	(2)	$-R \rightarrow -S$
P	(3)	$N \rightarrow S$
AA3.1	(4)	S
DC2.4	(5)	R.

At approximately this stage in the curriculum (depending on each student's individual rate of progress), a multiple-choice mode was available for use at the teletype terminals. Two inserted lessons used this multiple-choice mode for review and practice on logical vocabulary. One new rule, Double Negation (DN), was introduced by using the multiple-choice mode for direct instruction. The first inserted lesson contained 20 problems and the second lesson 19 problems.

Lesson 8 contained 18 problems having from one to three premises and required one-step through four-step derivations to derive the conclusions. Practice in applying the Double Negation Rule was emphasized. For example:

Derive:		B
P	(1)	$--(A \rightarrow B)$
P	(2)	A
DN1	(3)	$A \rightarrow B$
AA3.2	(4)	B.

The problems in Lesson 9 featured another new rule, Hypothetical Syllogism (HS). There were 21 problems in this lesson that required from one-step through five-step derivations. From one to three premises were provided for each problem. One problem required the use of an algebraic rule in its derivation. The rule of the Hypothetical Syllogism was applied in the following typical problem:

Derive:		$A \rightarrow D$
P	(1)	$A \rightarrow B$
P	(2)	$B \rightarrow C$
P	(3)	$C \rightarrow D$
HS1.2	(4)	$A \rightarrow C$
HS4.3	(5)	$A \rightarrow D$.

Lesson 10 contained 27 problems with from one to five premises that required from one-step to twelve-step proofs for solution. Many applications of the algebra rules were necessary for the problems in this lesson. Also, the Law of Addition, Form a Disjunction (FD), was presented. This rule permitted the student to type the second part of a disjunction formula. The underlined sections indicate work typed by the student. For example:

Derive:		$-S$
P	(1)	$S \rightarrow -(R \vee T)$
P	(2)	R
FD2	(3)	$R \vee (T)$
DC1.3	(4)	$-S$.

In Lesson 11 some of the 17 problems required derivations and some of the problems were presented in the multiple-choice mode. Those problems of the multiple-choice type reviewed the vocabulary and required the student to identify a certain type or part of a formula. The derivations contained from one to six premises with from two to twelve lines of rule applications for the solutions.

Lesson 12 combined both derivations and multiple-choice problems for the introduction of two new rules that applied the Commutative Laws. The first rule was called Commute Disjunction (CD), and the second rule was called Commute Conjunction (CC). There were 18 problems in this lesson; the nine derivations had either one or two premises and were one step or two steps in length. The Rule CD was applied as follows:

Derive: A v B
P (1) B
FD1 (2) B v (A)
CD2 (3) A v B.

Lesson 13 emphasized the combined use of algebra rules and logic rules. The 27 problems included both multiple-choice problems and derivations having one to three premises with as many as six lines of rule applications. The 16 problems in Lesson 14 followed the same format of combining multiple-choice problems with derivations that included the use of algebra rules.

Algebra

The algebra curriculum was presented in much the same format to the students as the logic curriculum, with the exception that rules were introduced in a notebook written in a programmed format. This approach was initiated because there was no multiple-choice mode available when the algebra program started.

Directions written into the program instructed the student when to read the introduction and when to solve the problems for a new rule in his notebook. The student then used the answer section in his notebook to check his work. The first two pages of the notebook included the rule names for both the logic and algebra programs and examples of their application. Each student had his notebook at the teletype terminal available for reference each day.

Lesson 1 contained 10 problems in which the student practiced the rule Number Definition (ND). (Each positive integer greater than 1 is defined as its predecessor plus 1. Thus $2 = 1 + 1$, $3 = 2 + 1$, etc.) This rule was printed with a prefix that indicated which number the machine was to present and define. For example:

Derive: $6 = 5 + 1$
6ND (1) $6 = 5 + 1$.

The underlined section shows the student's command to the computer.

Lesson 2 presented 15 problems that required the student to apply the Rule of Number Definition and then the new rule, Definition (D), that allowed the definition of a particular number to be substituted for (the name of) the number in a given number sentence. A prefix number in front of the rule abbreviation indicated the number that was to be replaced by its definition, and a postfix number indicated which *occurrence* of the number in the given sentence was to be defined. For example:

Derive:		$8 = ((5 + 1) + 1) + 1$
8ND	(1)	$8 = 7 + 1$
7D1	(2)	$8 = (6 + 1) + 1$
6D1	(3)	$8 = ((5 + 1) + 1) + 1.$

Lesson 3 contained 20 problems using both the Rule of Number Definition and the rule of Definition for two-step to four-step derivations. Lesson 4 provided further practice using the same rules for 15 problems that required three or four lines of proof.

In Lesson 5 a new rule, Commute Addition (CA), was introduced. To apply this rule to the previous line of the problem, a postfix number indicated which occurrence of the plus sign was used for the commutation. For example:

Derive:		$7 = 1 + 6$
7ND	(1)	$7 = 6 + 1$
CA1	(2)	$7 = 1 + 6.$

For the 20 problems in this lesson, both the Rule of Number Definition and the Rule of Definition were used continuously.

Lesson 6 contained 23 problems that required as many as four steps of proof. The three rules available for algebra proofs were used. Lesson 7 provided further practice with the same rules. The 22 problems required as many as seven steps for a solution. Lesson 8 extended the use of the same three rules. The 13 problems needed as many as eight lines of proof for the derivation.

In Lesson 9 a new rule, Associate Addition to the Right (AR), was introduced. The student typed a postfix number to indicate which plus sign was to be dominant after applying Associate Addition to the Right. For example:

P	(1)	$(4 + 3) + 1 = (4 + 3) + 1$
AR1	(2)	$4 + (3 + 1) = (4 + 3) + 1.$

There were 20 problems in this lesson that needed as many as five steps of proof for solution.

Lesson 10 provided practice with all rules that had been presented. There were 21 problems that required as many as seven steps of proof. Lesson 11 contained 11 problems that provided further practice with the same rules.

Lesson 12 contained a new rule, Inverse Definition (ID). This rule put a number in place of its definition. A postfix number was required to indicate which occurrence of a number's definition was to be replaced by the number. For example:

P	(1)	$5 + 1 = 5 + 1$
6ID2	(2)	$5 + 1 = 6.$

The postfix 2 indicates that the second occurrence of the definition of 6 is to be replaced. There were 20 problems in this lesson and some required as many as seven steps of proof for a solution.

In Lesson 13 there were 17 problems that needed as many as six steps for a derivation. Lesson 14 introduced a new rule, Associate Addition to the Left (AL). This rule allowed the students to reassociate numbers to the left using the same format as Associate Addition to the Right. There were 17 problems in this lesson.

Thus, in these 17 algebra lessons a total of six algebraic rules of inference were introduced. The introduction of these rules gave the students experience with the sort of mathematical inferences that are widely used in elementary algebra and that are rather different from the rules of sentential inference.

Computer-assisted Arithmetic Drill and Practice

A computer-assisted drill-and-practice program for elementary-school arithmetic instruction was included in the curriculum for this class during 1966-67. The program is described in detail in Suppes, Jerman, and Brian (1968). These students drilled on fourth-grade problems that were prepared at five different levels of difficulty. This curriculum was written in concept blocks, like addition or subtraction, that required seven days of work. The first lesson of the concept block contained a pretest for the material included in that concept block with problems representing all five levels of difficulty. The student's performance on this pretest determined the level of difficulty of his next drill. Each day's performance within the concept block determined the level of difficulty of his following drill. On the seventh day of the concept block a posttest designed like the pretest completed the concept block. The next concept block started with a pretest for the new material. In addition to the particular concept block the student was studying, approximately five problems were reviewed according to his individual need determined by his lowest posttest score on previous concept blocks. Each time he reviewed a concept block his new posttest score for that block was determined by his latest performance. A description of the concept blocks is given in Table 4.

Daily Problem Sets

During the previous school year these children drilled on arithmetic problems for a few minutes each day in the classroom. The same procedure was followed this year until the teletype drill-and-practice program began operating at the end of November. There were usually 10 to 15 problems to be solved each day. After the individualized teletype program started, a problem set with seldom more than five problems was presented daily for the remainder of the year. The short problem sets were used for group discussion emphasizing different possible solutions to the given problems. Special formats required for responding to the teletype drill-and-practice program were given during these discussion periods.

The problem sets included review of all arithmetic operations presented in various formats in the *Sets and Numbers* texts and other current texts. Number relations, units of measure, fractions, word problems, and dictations were included in these problem sets. Detailed descriptions and results of these problem sets may be obtained from the American Society for Information Sciences.[2] Mathematical games and puzzles were also brought to the class by the students and the teacher.

Group Composition

In September, 1966, the group was composed of 32 children, 15 girls and 17 boys, who were distributed among four schools in subgroups of 5, 6, 9, and 12. The oldest child was 9.5 years old and the youngest child was 8.5 years old. With

[2]Order NAPS Document 656 from ASIS National Auxiliary Publications Service, c/o CCM Information Sciences, Inc., 909 Third Avenue, New York. 10022; remitting $1.00 for microfiche or $3.00 for photocopies.

the exception of one student who had been promoted from third grade to fifth grade, the students were in fourth grade.

In June, 1967, 30 students, 14 girls and 16 boys, remained in the group after 2 students moved from the school district.

Class Procedure

The students attended four neighboring elementary schools, but all of the students met at one of those schools for their mathematics instruction this year. School buses provided by the school district transported the students from three of the schools to the fourth school. None of the neighboring schools was more than a 10-minute bus ride from the school used for the classes. Children from two schools comprised each of the two class sessions that were taught by the same teacher (Mrs. Ihrke).

In September, 1966, there were 17 members in one class session and 15 members in the other; in June, 1967, there were 16 members in one class session and 14 members in the other. The students' total mathematics instruction was contained in five daily class sessions that lasted from 45 to 55 minutes.

Two adjoining schoolrooms were used for each session. One of the rooms was used as a classroom and the other room housed the 8 teletype machines used for the computer-assisted instruction. Each day in the classroom the students were asked to solve a set of problems that were appropriate for fourth graders. Again this year each student worked individually through the *Sets and Numbers* textbooks. Each student corrected all errors in his previous day's work before he began any new problems. Occasionally the classroom setting was used for group activities that included problem-solving and drill presented as games or puzzles. Games and puzzles for individual use were available during designated class time and for use at home.

The room adjacent to the classroom contained 8 teletype machines connected by telephone lines to a PDP-1 computer at Stanford University. The teletypes were spaced several feet apart against three of the walls in the room. This arrangement allowed the students to work independently with little distraction from each other.

Two separate programs were presented as computer-assisted instruction. First was the drill-and-practice program for arithmetic skills that started during the last week of November, 1966. Each drill provided from four to ten minutes of practice in basic arithmetic skills. Often students were allowed to do more than one arithmetic drill when class time permitted.

Second was the sentential logic which started the second week in December, 1966. Each student worked from six to ten minutes per day solving logic problems. In conjunction with the logic program, the algebra program started the fourth week of January, 1967. After that date the students automatically alternated between the logic program and the algebra program each day and continued with the separate arithmetic drill program. Each student worked approximately fifteen to twenty minutes in the teletype room daily. The classroom teacher assigned additional arithmetic drills to students who had been absent or needed supplementary practice. When the logic program was initiated a staff member was available in the teletype room to answer the students' logic questions and to direct machine problems to the

Institute's computer staff. As machine problems became less frequent and improvements were made in the logic program, it was not necessary for someone to be in the teletype room with the students every day. The classroom teacher supervised the students in the teletype room with the aid of a one-way window between the two rooms.

RESULTS

Sets and Numbers Texts

For the fourth year the results show considerable differences in the number of problems solved and the error rates for individual students. Table 1 presents the

TABLE 1. NUMBER OF PROBLEMS COMPLETED AND ERROR RATE FOR *Sets and Numbers* TEXT MATERIAL, TOP FOUR AND BOTTOM FOUR STUDENTS

Student	Books	Number of problems completed	Percentage of problems in error
1	5, 6	9196	5.86
2	4A, 4B, 5	8836	3.97
3	4A, 4B, 5	8655	6.15
4	4A, 4B, 5	7206	5.37
27	4A, 4B	3133	3.57
28	4B, 5	2921	6.71
29	4B	2117	7.46
30	4B	1578	7.29
Mean (30 students)		5031.70	5.88

results of the four students who completed the greatest number of problems and the four students who solved the least number of problems in the *Sets and Number* texts. Only one of the students who completed the most number of problems in 1965-66 completed the most number of problems in 1966-67. One of the students who completed the least number of problems in 1965-66 completed the most number of problems in 1966-67. The 30 students who participated during the entire school year were considered for the results in the *Sets and Numbers* work. The mean percentage of errors was 5.88 for the 1966-67 year, which was an increase of .44 per cent over 1965-66. In addition to *Sets and Numbers* problems, arithmetic drills, logic and algebra problems, and daily problem sets were solved by each student.

Table 2 shows detailed results of the students' work in the *Sets and Numbers*

TABLE 2. DETAILED RESULTS OF STUDENTS' WORK IN *Sets and Numbers* TEXTS

Chapter	Book section	Mean no. problems completed	Range of problems completed	Mean % problems in error	Range of % error	*N*
	Book 3B					
4	Commutative	73.00	73	0.00	0	1
5	Associative laws	342.00	276-408	1.59	1.45-1.72	2
6	Shapes and sizes	8.00	8	0.00	0	1
7	Distributive law for multiplication	193.67	116-340	14.90	3.45-33.60	3
8	Lines of symmetry	70.00	18-122	0.82	0-1.64	2
9	Word problems	129.67	115-137	16.51	8.70-9.45	3

TABLE 2. (Continued)

Chapter	Book section	Mean no. problems completed	Range of problems completed	Mean % problems in error	Range of % error	*N*
	Book 4A					
1	Review of sets, addition, and subtraction	195.43	59–219	5.97	1.38–12.39	7
2	Review of geometry	44.00	44	12.66	2.27–22.73	7
3	Addition and subtraction	363.00	64–475	8.98	3.36–13.47	9
4	Geometry	84.18	37–102	12.86	4.90–23.08	11
5	Review of multiplication and division	280.09	238–309	3.95	1.26–11.94	11
6	Laws of arithmetic	797.41	403–896	2.99	1.12–7.47	12
7	Shapes and sizes	45.54	16–48	8.97	0–31.25	13
8	Distributive law for multiplication	669.12	97–910	3.91	0.93–11.34	17
9	Applications	380.59	285–367	6.84	3.10–17.92	17
10	Distributive law for division	466.05	200–540	5.26	1.38–13.55	19
	Book 4B					
1	Circles and lines	85.81	44–90	3.69	0–25.56	21
2	Subsets and less than	58.79	15–62	8.46	0–22.58	19
3	Fractions	350.05	125–375	4.75	0.40–14.67	19
4	Division	27.00	27	0.00	0	1
5	Average	54.37	26–60	15.93	3.85–29.31	19
6	The number line	272.00	93–333	10.29	1.83–33.60	19
7	More about geometry	109.35	36–127	2.99	0–9.45	17
8	Review of the commutative, associative, and distributive laws	496.20	94–584	4.80	1.71–9.62	15
9	Word problems	141.50	125–146	12.66	6.16–26.03	14
10	More about sets	162.64	130–187	8.24	2.53–22.78	14
11	Logic	94.00	94	0.00	0	1
	Introduction to long division	1,176.67	987–1474	4.52	0.75–9.53	30
	Book 5					
1	Sets	179.05	66–195	7.24	2.06–13.64	20
2	Laws of arithmetic	455.94	180–488	5.55	1.84–10.86	18
3	Multiplication and division	498.94	111–708	10.84	2.26–19.54	17
4	Geometry	167.08	6–209	3.76	0–7.18	12
5	Fractions	515.25	444–529	4.69	1.91–7.75	8
6	More divisions	297.25	80–528	7.35	0–13.13	8
7	Measuring	320.50	235–345	14.00	8.41–17.70	6
8	More fractions	459.50	144–569	6.47	2.99–10.37	6
9	Space figures	150.00	150	4.17	2.00–10.67	4
10	Systems of numeration	432.75	420–437	6.39	3.81–9.38	4
11	Integers	209.33	134–246	4.91	2.99–5.28	3
12	A coordinate system	213.00	9–319	4.37	0–9.97	3
13	Graphs	156.00	156	5.77	5.13–6.41	2
14	Mathematical sentences	240.50	145–336	6.37	4.46–8.28	2
15	Using fractions	436.00	436	12.39	0	1
16	Decimal fractions	396	396	6.28	0	1
17	More geometry	212	212	4.25	0	1
18	Logic	176	176	2.27	0	1
19	Probability	57.76	26–94	7.42	0–46.81	29
20	Graphs and functions	129	129	12.40	0	1
21	More about sets	119.50	42–197	6.11	5.08–7.14	2
	Book 6					
1	Sets	240		4.17	0	1
2	Laws of arithmetic	438		3.65	0	1
3	Fractions	331		5.44	0	1
4	Geometry	201		3.98	0	1
5	Factors and multiples	160		2.50	0	1

texts. Included are chapter descriptions, the mean number of problems completed, range of the number of problems completed, the mean percentage of problems in error, range of the error rates, and the number of students who solved problems in each section. Differences in the number of problems completed occurred when students started or finished the 1966-67 school year within a given book section or when the teacher suggested that a student solve a certain portion of a given section. Approximately two years of the *Sets and Numbers* curriculum separated the fastest and the slowest students at the end of 1966-67. This is a greater spread than existed at the end of 1965-66. Table 3 shows each student's class ranking for the *Sets and Numbers* text work and other areas of the curriculum.

TABLE 3. CLASS RANK FROM 1 TO 30 FOR PERFORMANCE IN THE CURRICULUM AREAS

Student Number	*Sets and Numbers* problems completed	Grade placement in *Sets and Numbers*	Mean percentage of errors in *Sets and Numbers*	Logic problems completed	Algebra problems completed	Logic and algebra problems completed	Mean percentage correct, teletype drill posttests
1	15	26-27	28	9	16	13-14	29
2	29	18	22	17-18	14	16	28
3	1	1	13-14	4-5	4	2-3	6
4	13	29-30	7	7	18-19	9	24
5	21	17	29	19	17	18	26
6	23	5-6	27	8	5	8	17
7	16	12-15	6	3	1	1	11-13
8	10	19-20	1	2	9	5	1
9	7	21-24	12	13-15	22	21	19
10	18	28	3	12	11-13	11-12	11-13
11	28	12-15	18	25	18-19	22	21-22
12	26	12-15	24	23	21	23	20
13	8	3	2	11	2	6	2
14	11-12	29-30	11	30	24	29	25
15	2	7-8	8	13-15	3	10	7
16	22	12-15	13-14	17-18	7-8	13-14	8
17	6	21-24	30	26-27	25	26	15
18	3	2	16	10	23	19	27
19	30	25	21	29	30	30	30
20	25	21-24	19	26-27	27	27	23
21	17	26-27	25	28	26	28	21-22
22	24	5-6	23	1	10	2-3	4
23	9	4	17	4-5	6	4	3
24	20	7-8	26	20	28	24	9-10
25	4	9-11	10	16	7-8	11-12	11-13
26	11-12	9-11	5	6	11-13	7	14
27	14	19-20	9	13-15	15	15	16
28	19	9-11	20	24	29	25	9-10
29	5	16	15	21	11-13	17	18
30	27	21-24	4	22	20	20	5

Logic and Algebra

The number of logic problems solved by the students ranged from 162 to 285 problems. The mean number solved was 220.5 problems. From 130 to 244 algebra problems were solved with 194.7 the mean. Some further details about the results of the logic program are reported in Suppes and Morningstar (in press).

Computer-assisted Drill and Practice

Table 4 shows the mean class performance on pretests and posttests for the

TABLE 4. DRILL-AND-PRACTICE PRETESTS AND POSTTESTS

Concept block	Description	Mean % correct Pretest	N	Mean % correct Posttest	N
1	Addition	78.9	30	87.3	30
2	Subtraction	84.4	30	94.7	30
3	Subtraction	87.2	30	90.7	30
4	Addition	94.8	30	96.0	30
5	Addition, Subtraction	80.0	30	87.3	30
6	Measures	81.5	30	89.4	29
8	Mixed drill	89.3	29	89.8	29
9	Laws of arithmetic	78.3	29	91.5	30
10	Division	91.1	30	95.8	30
11	Multiplication	92.0	30	96.7	30
12	Fractions	60.7	30	72.2	30
13	Mixed drill	74.6	30	89.2	29
15	Laws of arithmetic	85.4	30	87.7	29
16	Fractions	76.1	29	87.0	12
22	Mixed drill	88.6	29	92.8	29
24	Division	75.4	30	89.6	30
25	Mixed drill	96.1	30	96.8	30
26	Mixed drill	96.5	29	97.1	30

various concept blocks. The data are based on the performances of the 30 students who participated in the class throughout the year. Block 16 was the last concept block for most of the students, and some of the students were not able to finish that block before the end of the year. For certain concept blocks, complete data for 30 students were not available. Concept Blocks 22, 24, 25, and 26 were special blocks inserted into the curriculum during the school year. Figure 1 shows the students'

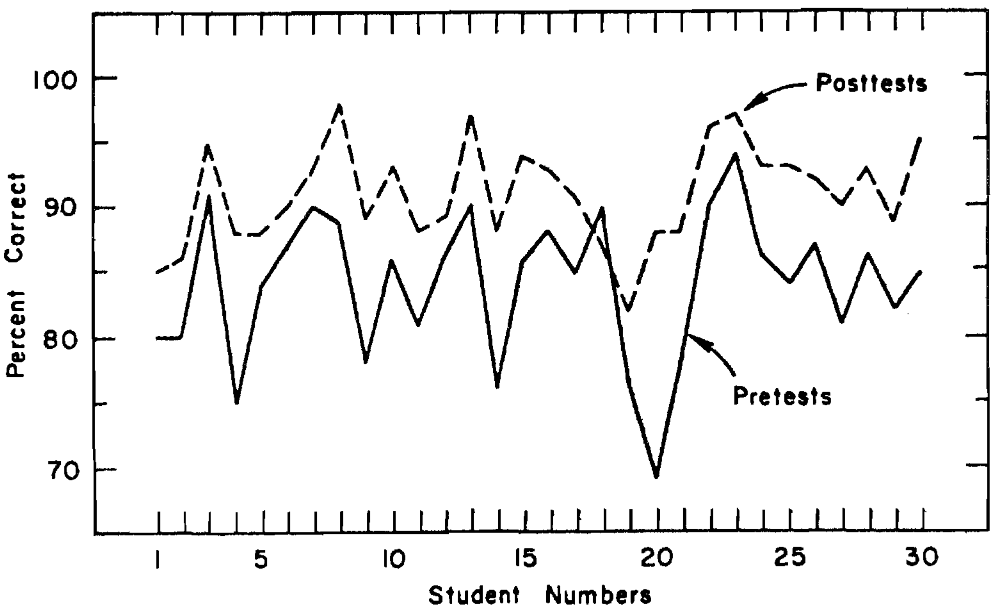

FIG. 1. Students' mean scores on drill-and-practice tests.

mean performance on pretests and posttests. One student showed an average decline on posttests compared with pretest performances. All other students showed definite improvement for the combined performances of tests.

Figure 2 shows the spread in performances on pretests and posttests. Most of the scores are clustered above 80 per cent correct. Again, the improvement on posttests is a general rule.

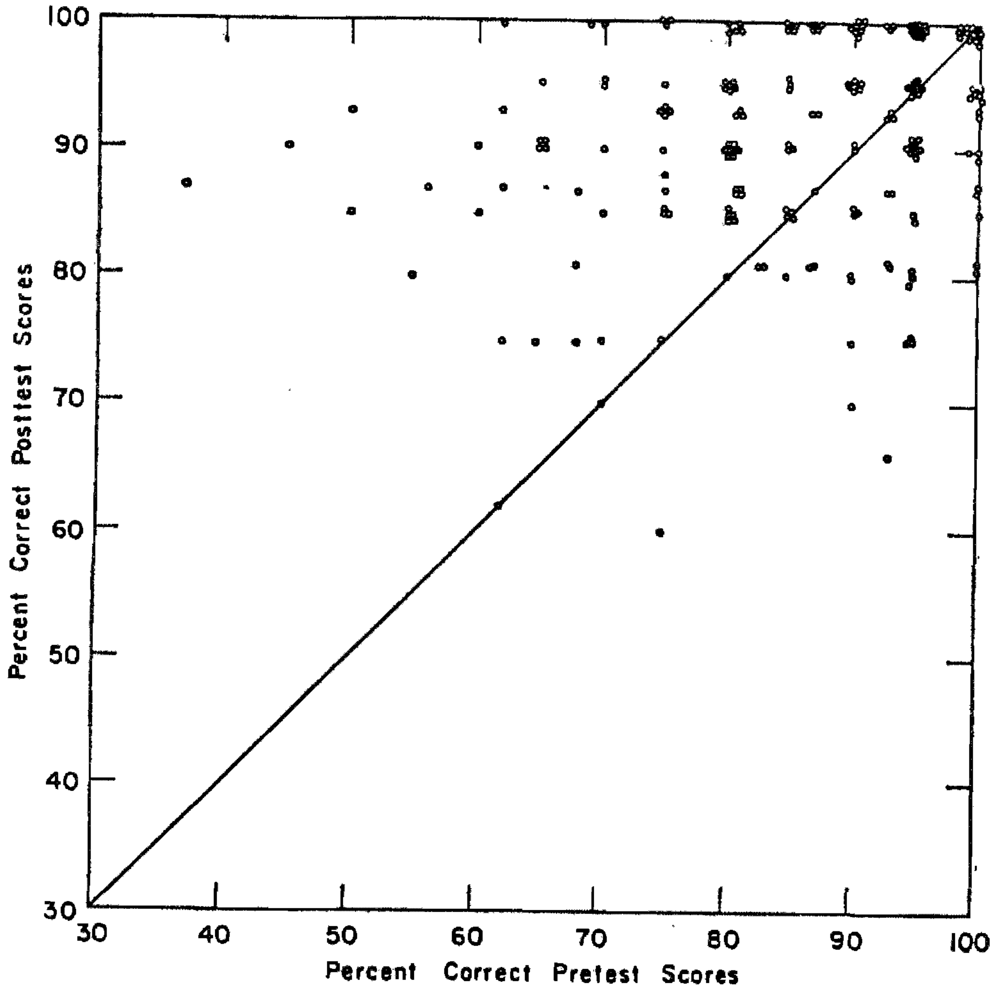

FIG. 2. Pretest and Posttest performance for each student on each concept block.

Standardized Tests

Table 5 shows the results of the standardized tests administered during the school year. The school and College Ability Test (SCAT) measured the students' verbal and mathematical abilities and the Stanford Achievement Tests (SAT) measured the students' progress in mathematics. A wide range occurred for each section of each test. Percentile ranks were based upon the grade level of the students at the time the tests were administered.

TABLE 5. STANDARDIZED TESTS

Test	Section	Mean percentile	Percentile range	Date
School and College Ability Test	Verbal	92.7	31.0-99.9	Fall, 1966
	Quantitative	97.5	70.6-99.9	Fall, 1966
	(total)	97.0	65.8-99.9	Fall, 1966
Stanford Achievement Test, Form W	Computation	65.9	34-98	Fall, 1966
	Concepts	96.2	66-99	Fall, 1966
	Applications	92.5	58-99	Fall, 1966
Stanford Achievement Test, Form X	Computation	82.4	28-99	Spring, 1967
	Concepts	96.8	74-99	Spring, 1967
	Applications	94.8	77-99	Spring, 1967

The comparison group which is one year younger and was selected by the same instruments as the accelerated group completed its third year of mathematics (see Sears, Katz, & Soderstrum (1966) for details about the comparison group). In the spring of 1967, these students were given the Stanford Achievement Test, Arithmetic Primary II. Some grade-level results for the accelerated class and the comparison group, based on the different forms of the Stanford Achievement Tests for the spring of 1967, follow. The accelerated group had a mean score of 6.4 years for computation (1.5 years above grade level) and a mean score of 7.7 years for concepts (2.8 years above grade level). The comparison group had a mean score of 3.9 years for computation (at grade level) and a mean score of 5.5 years for concepts (1.6 years above grade level). One student in the accelerated class was below grade level in computation (−0.6 year). In the comparison group 14 students were below grade level in computation (range −1.6 to −0.2 years) and one student was below grade level in concepts (−0.8 years).

A study of the personality and social development of both groups of students was continued under the direction of Dr. Pauline S. Sears, and the results will be published elsewhere.

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