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ACCELERATED PROGRAM IN ELEMENTARY-SCHOOL MATHEMATICS—THE THIRD YEAR[1]

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This report describes the third year (1965-66) of a longitudinal study of the accelerated program in elementary-school mathematics conducted by the Institute for Mathematical Studies in the Social Sciences. A description of the first year of the study, including details of the procedures by which the students were selected, was reported in Suppes and Hansen (1965). The second year was reported in Suppes (1966). The present report was written to be as homogeneous as possible with the earlier ones. In the second section, we describe the curriculum content of the third year. The third and fourth sections contain a brief description of the class composition and class procedure. The fifth section, on results, reports the systematic behavioral data collected. The tables and figures are similar to those presented in the two previous reports, but there are some changes in presentation which reflect changes in curriculum and teaching procedure. For example, no meaningful comparisons can be made between this year and previous years in rate of problem acquisition in the *Sets and Numbers* curriculum because of the varying amount of class time allotted to *Sets and Numbers* work. This year less class time was devoted to these texts, and a greater amount of time was spent on special enrichment materials.

The 32 children who participated in the 1965-66 program were bright third graders in the third year of an accelerated program in elementary-school mathematics. These facts should be considered when reading the descriptions of the curriculum.

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Curriculum Description

Sets and Numbers

During the academic year 1965-66 the students continued to work individually on the *Sets and Numbers* text material in Books 3A, 3B, 4A, 4B and 5A (which were preliminary editions of Books 3, 4 and 5). Chapter descriptions are included in Tables 2 and 3. The following topics are presented in the *Sets and Numbers* texts: numbers and numerals, place value, addition, subtraction, multiplication, division, sets, fractions, measurements, geometry, problem solving, equations and inequalities, graphs and charts, laws of arithmetic, and logic. Since the *Sets and Numbers* texts constituted the students' basic mathematics program, the teachers carefully observed each student's individual progress. Students who found certain concepts difficult received additional practice on those concepts, while students who made few errors were allowed to skip repetitious problems as directed by the teachers.

Logic

The logic curriculum for the 1965-66 school year was an extension of the program initiated during the summer session of 1965. Mimeographed lessons were copied from the lesson books prepared for use on the cathode-ray-tube display terminals at the Stanford University Computer-based Laboratory for Learning and Teaching. Instead of maintaining the two different approaches involving separate tracks for problems written in English and problems expressed in symbolic logic (as discussed in the report for the second year of the study), a combination of the two forms was presented throughout the year.

The first class sessions in the fall were used to review the material from the summer session and to familiarize all the students with the various formats used. Since the number of problems per lesson ranged from 9 to 58, the number of class sessions for each lesson varied accordingly. Usually between two and four problems were completed per day.

Lesson 5, containing 40 problems, was the first one used by all students during the school year. This lesson began with derivations written in English that required the student to choose the correct answer from among three possible responses. The answer represented the last step in a derivation. For example:

Premise 1. If Jim has a vacation, then he goes to camp.
Premise 2. If he goes to Camp Arrow, then he sleeps in a tent.
Premise 3. He goes to Camp Arrow.
Premise 4. If he goes to Happy Camp, then he sleeps in a cabin.
5. A. He sleeps in a cabin.
B. He sleeps in a tent.
C. He goes to Happy Camp.

The student was asked to circle his answer and to note which two premises had determined his choice. As the student progressed through the lesson, he was to write the correct conclusion without the benefit of multiple-choice answers. Two-step derivations applying *modus ponendo ponens,* called the IF rule, were included in both English-sentence and symbolic representations after some practice in translation from English to a symbolic format. The student was required to list the line numbers of premises he used for each step of his proof. For example:

Derive: J
Premise 1. If T then B
Premise 2. L
Premise 3. If B then J
Premise 4. T
___, ___ 5. ______
___, ___ 6. ______

The first 13 problems in Lesson 6 served as a review of two-step derivations (in both English and in symbolic format) requiring application of *modus ponendo ponens* as presented in the previous lesson. Next, a new symbol, '→', was introduced to represent the sentential connective 'if........, then........'. Practice was provided in translation from the English format to the symbolic format in addition to the actual derivations required. For example:

'S' stands for 'It is Saturday'.
'N' stands for 'There is no school'.
'C' stands for 'We go to the city'.

Derive: We go to the city.
Premise 1. It is Saturday.
Premise 2. If it is Saturday, then there is no school.
Premise 3. If there is no school, then we go to the city.
___, ___ 4. ______
___, ___ 5. ______

Derive: C
Premise 1. ______
Premise 2. S → ______
Premise 3. N → ______
___, ___ 4. ______
___, ___ 5. ______

Lesson 7 consisted of 20 English-sentence and symbolic problems that involved one-step or two-step derivations using still only *modus ponendo ponens*. Lesson 8 introduced the symbol '¬' for the sentential connective 'not'. A variety of one-step and two-step proofs in both English and symbolic form were included. For example:

Derive: P
Premise 1. ¬S → P
Premise 2. R → ¬S
Premise 3. R
___, ___ 4. ______
___, ___ 5. ______

In Lesson 9 longer proofs were required to solve the problems. Both English and symbolic formats were used for the problems, which involved from two steps to five steps.

Lesson 10 introduced the Rule of Conjunction, called the AND rule in class, which allowed the students to conjoin two sentences. Both English and symbolic formats were presented in this lesson.

In Lesson 11 the students met the first problems that required more than one rule to solve a problem. Both *modus ponendo ponens* (IF) and the Conjunction Rule (AND) were necessary for the proof of some of the problems. For example, consider this problem with four premises:

Derive: E
Premise 1. $X \rightarrow Y$
Premise 2. X
Premise 3. $(X \,\&\, Y) \rightarrow Z$
Premise 4. $(X \,\&\, Z) \rightarrow E$

Lesson 12 contained 12 problems that were composed of mathematical sentences. The solutions to the problems involved proofs of from one to five steps, based on *modus ponendo ponens* and the Rule of Conjunction. For example:

Derive: $A - B = A$ and $B - A = B$
Premise 1. $A = [\ \ ]$
Premise 2. $B = [1, 2]$
Premise 3. If $A = [\ \ ]$ and $B = [1, 2]$, then $A \cap B = [\ \ ]$
Premise 4. If $A \cap B = [\ \ ]$, then $A - B = A$
Premise 5. If $A \cap B = [\ \ ]$, then $B - A = B$

In Lesson 13 the Rule of Simplification was introduced for inferring one member of a conjunction. In the classroom the rule was designated as 'L &' or '& R' to correlate with the left or right member of a conjunction. The lesson involved both the Rule of Simplification and the Rule of Conjunction for English sentences and symbolic format.

The problems in Lesson 14 required the use of *modus ponendo ponens*, and the Rules of Conjunction and Simplification. The proofs contained from two to six lines. Both English and symbolic formats were presented.

Lesson 15 contained nine problems written in mathematical sentences that required two to five steps, using the three rules of inference already mentioned. In Lesson 16 more practice was provided in both English and symbolic format. The problems required from one-step to five-step proofs.

In Lesson 17 the emphasis was on the student's selection of dominant connectives. The first part of the lesson required the student to select the dominant connective from symbolic sentences. Next, the student solved problems using the three mentioned rules of inference, after choosing the correct dominant connectives. The third section of the lesson required the student to translate from English format to symbolic format and to select the dominant connective.

Lesson 18 provided additional practice with proofs containing from one to six steps, using both English and symbolic formats.

In Lesson 19 *modus tollendo tollens* was introduced. In class it was called the OR rule. The first part of the lesson involved only one-step to three-step proofs using this one rule of inference. The final problems in the lesson required use of all four rules of inference, but of course, not necessarily all four in each problem. For example:

Derive: F & P
Premise 1. $W \vee F$
Premise 2. J & P
Premise 3. $\neg W$

Lesson 20 provided practice with the four rules of inference. The problems had from two to five steps, including symbolic format, English sentences, and mathematical sentences. For example:

Derive: $m = 2$ and $m < n$
Premise 1. If $n > 2$ or $n < 2$ then $n \neq 2$
Premise 2. $n > 2$ or $n < 2$
Premise 3. Either $m = 2$ or $n = 2$
Premise 4. $m < n$

There were 21 of the 32 students who completed work through Lesson 20; the remaining 11 students finished through Lesson 17. Most of the students did not consider the material difficult. The teachers observed that most errors were related to errors in reading rather than lack of comprehension of the logic involved. Most errors were quickly corrected by a student when he was asked to read the problem orally, describe the premises, and determine what rules of inference could be applied to each of the premises.

Three of the students who found the class work difficult received additional instruction and practice at the Stanford Computer-based Laboratory. The students were highly successful in their work at the Laboratory where the machine was programmed to check each response a student made.

Symmetry

The material on finite symmetry groups was an introduction both to group theory and the perception of symmetry. This topic was chosen because of its intuitive appeal, and because the students had already done a certain amount of geometry. They were familiar with many of the geometric properties of such simple geometric figures as rectangles, squares, and triangles. They were familiar with determining lines of symmetry from previous work in the *Sets and Numbers* texts.

An introduction to the symmetry-groups material was presented briefly in Suppes (1966) in the discussion of the second-year curriculum. This report will discuss the contents of each section of that material which the students have studied.

Section A—Lines of Symmetry. A short discussion of the difference between a figure and a region was presented at the beginning of the section. Next the students reviewed the definition of a line of symmetry. The children were reminded that if both parts of a region exactly matched when the region was folded along a given line, that line was a line of symmetry. The mirror test for symmetry was also demonstrated.

Practice was given in identifying and drawing lines of symmetry. Straightedges and pencils were the only drawing tools used by the students. Although accuracy was not expected, the students used the folding test to check their own work. The teachers stressed the convention of numbering the lines of symmetry in a clockwise direction. Next the students were asked to match regions that had the same number of lines of symmetry. Many exercises did not require the student to fold a region or to draw lines of symmetry. If he made errors the teacher asked him to fold or draw lines of symmetry to check his answers.

Section B—*The Positions.* In this section, regions were discussed as if they had frames that were in a stationary position. Thus the region was movable, but its frame was not. Physical models of regions with frames were available for the students. Since the regions that were represented had various numbers of lines of symmetry or no lines of symmetry, the regions could fit into the frames in many different ways. The students were asked to count the number of ways each region would fit in its frame in both face-up positions and face-down positions.

Next the student was asked to consider a region pictured on paper. If he cut out the region the remaining paper was called the frame of the region. The student could manipulate this region in the same manner used for the cardboard models. To differentiate between the face-up position and the face-down position, the face-up position was noted by placing an "O" on one corner and the face-down position was noted by an "X" on the back of the same corner. After working several exercises in which comparisons were made between the number of face-up positions, the number of face-down positions, and the number of lines of symmetry, most of the students realized that the number of lines of symmetry and the number of face-down positions were the same.

Next the student was asked to picture all possible face-up and face-down positions of a region by putting the correct symbol, "O" or "X", in the appropriate place. When the student found it difficult to picture all of the positions for a region, he could cut out the region or use another model to solve the problem.

Section C—*The Motions: Flips.* In this section the term "flip" was introduced as a name for the single motion that was used to change a region from a face-up position to a face-down position or to change a region from a face-down position to a face-up position. The student was instructed to flip a region by holding a given line of symmetry and flipping the region over it. Flips were named according to the number of the line of symmetry that was used. Thus a flip over symmetry-line 2 was called "Flip 2" and abbreviated "F2". Most of the children easily related the number of possible flips for a region to the number of lines of symmetry for the region. Next the child was given a picture of a region and asked to picture the new position that would result after he performed a specified flip. Again, the student could check his work by cutting out the region and performing the required manipulation.

Section D—*The Motions: Rotations.* In this section the students were introduced to rotations which could be used to place a region in a different position in its frame, i.e., the orientation-preserving rigid motions. The students were instructed to make all rotations in a clockwise direction. Rotations were named according to the number of "notches" the region was turned. Thus a turn of two notches in a clockwise direction would be read "Rotation 2" and abbreviated "R2"

The identity motion of Rotation Zero, RO, was performed by taking a region out of its frame and placing it back in that identical position.

When the student was shown two pictures of the same region, he was asked to tell what rotation had been performed on the region that would explain the difference between the first and second pictures. Next he was shown a picture of a region and given a specific rotation that was to be performed on the region. He was asked to picture, by marking either an "O" or an "X" on the drawing, the new position of the region.

Next the children were asked to list and picture all possible motions for a given region. After noting this information for several different regions, the term "group" was introduced. A group for a region was defined as the set of all the motions for the region. After finding the groups for several different regions, the children were asked to match regions that had the same group.

Section E—*Adding the Motions.* In this section the children continued performing motions that required flips or rotations. Next the student was given a region and asked to make one motion and then another. For example: do Rotation 1, then do Flip 3. The notation would be R1 + F3 for this problem.[2] There were also mixed exercises requiring the students to perform (in imagination) both flips and rotations and to draw the results by picturing the new position.

The next step involved determining what single efficient motion would have produced the same result. For example, R1 + F3 = F2 (see footnote 2). When the order of adding the two motions did not change the result, the two motions were said to commute for the region. The children were asked to determine which motions would commute for various regions.

Plans have been made for the students to continue working on this material during the school year of 1966-67. Most of the students enjoyed this work very much and did not find the concepts difficult.

Daily Drills

In addition to the daily work in the *Sets and Numbers* texts, a daily drill program was established to insure frequent practice on basic arithmetical skills. The students were asked to perform a written drill almost every day. Although the teachers adjusted the classroom procedure according to the length and difficulty of the drills, the students' performance, and the amount of class time available, the sequence of the drills was the same for each class.

Most of the drills presented basic arithmetic concepts that were appropriate for third-, fourth- or fifth-grade students. Each drill consisted of from 4 to 30 problems, with 20 being by far the most common number.

A detailed description of each drill is in the Appendix of this report. The various formats used included the following. The exercises in addition were of the type $a + b = c$, in horizontal and vertical formats; $(a + b) + c = d$, horizontal format; $a + b + c + d = e$, vertical format; and $a + b = c + d$, horizontal format. The position of the variable or blank changed in the problems.

For the subtraction problems, the formats included $a - b = c$, horizontal and vertical formats; $a - b = c - d$, horizontal format; and $(a - b) - c = d - e$, horizontal format. The position of the variable changed in the problems.

[2] The plus sign in this notation should be enclosed in a circle but is not, due to lack of typeface.

The formats used for the multiplication problems were $a \times b = c$, horizontal and vertical formats; $c = a \times b$, horizontal format; $(a \times b) + c = d$, horizontal format; and 2-digit numbers times 1-digit numbers in a vertical format. The position of the variable changed in the problems.

Mixed drills that included several different operations in the same set of problems were frequently used. The students also compared numbers by choosing the appropriate symbol, $<$, $=$, or $>$. The mixed drills often included simple division facts. Word problems were another form of mixed drill. The students solved many problems which were in equation form. Some of the word problems asked the student to write the equation necessary for solving the problem.

The students performed comparisons in most of the fraction drills. The format was a____b and the students filled in the blank with $<$, $=$, or $>$. In the first problems the students compared proper and improper fractions to 1. Next the students compared improper fractions to whole numbers greater than 1. Then the students compared two different fractions whose denominators were the same. The children did some renaming of fractions to find equivalent fractions with different denominators. Finally, the students compared two different fractions whose denominators were not the same.

In addition to the written drills described above, the teachers occasionally supplemented classroom activity with arithmetic games and dictation. The dictation was particularly helpful when the teachers introduced numbers, which had at least 5 digits and required zeros in some digits.

Group Composition

The group was composed of 32 children, 15 girls and 17 boys, who were distributed among four schools in subgroups of 5, 6, 9, and 12. The subgroups of 5 and 6 members met as one class at one of the schools.

All of the children were third-grade students. In September, 1965 the oldest child was 8-1/2 years old and the youngest was 7-1/2 years old.

Class Procedures

Although the children in the program represented four schools, the students were organized into three separate classes, with the children from two of the schools meeting together. Each class met at one of the schools from 30 to 45 minutes five days a week; this period was the entire time allotted for the students' mathematics program.

Each day the students' work was divided among several phases of the curriculum. A written arithmetic drill was performed daily by each child. The remaining class time was used for individual work in the *Sets and Numbers* texts, logic, and geometry. Again this year each student progressed through the *Sets and Numbers* texts at his own rate. Each day students began work in the *Sets and Numbers* texts by correcting mistakes made the previous day before beginning new material. The logic exercises were checked during class discussion after the students had worked the problems independently.

The 26 children who attended the summer session conducted during July, 1965 studied specially prepared material on finite symmetry groups. Since the students were unable to complete the material during the short four-week course, the subject matter was continued during the school year. While the children who did not attend the summer session were allowed additional class time for working on the geometry material that had been presented, those children who had attended the summer session only reviewed the summer work and continued in the later chapters of the work. Although the students in each class worked individually, all students were asked to finish the preceding section before each new section was introduced to the class as a group. Emphasis was placed on class discussion of the material and manipulation of objects to illustrate new concepts.

Results

Sets and Numbers Texts

The results this year again show great variation in acquisition and error rates for individual students. Table 1 indicates the number of problems completed and error rates for the top four and bottom four students. Generally, the students

TABLE 1. NUMBER OF PROBLEMS COMPLETED AND ERROR RATE FOR *Sets and Numbers* TEXT MATERIAL, TOP FOUR AND BOTTOM FOUR STUDENTS

Student	Books	Number of Problems Completed	Per Cent of Problems in Error
1	4A,4B	6262	2.97
2	4A,4B	5435	2.55
3	4A,4B	4931	3.40
4	4A,4B	4789	6.13
29	4B,5A	1159	13.97
30	3B,4A	996	14.85
31	3B	743	8.34
32	4A	717	10.04
Mean (32 students)		2695.3	5.40

with a larger number of problems completed and a lower error rate were the students who were working in the more advanced text material, although Student 29 is a clear exception to this generalization. As would be expected, the slower students in the *Sets and Numbers* texts were also the ones who worked slowly in other phases of the curriculum. Also some of these children had not participated in the 1965 summer session.

For the 32 children the mean number of problems completed was 2695.3 and the mean number of problems in error was 145.7, with the mean per cent of error being 5.4. Last year there was a 4.5 mean error percentage based on the students' work in *Sets and Numbers*, Book 3B. The 1964-65 error rate was an increase of 1.7 per cent over the previous year, 1963-64, which was 2.8 per cent. The pattern of increasingly high mean error percentages in part is a reflection of the increasing complexity of the text material, and is ordinarily encountered in elementary-school mathematics programs. The rather striking difference between the number of problems completed by the top four and the bottom four students must be interpreted in the context of the fact that essentially all the students completed the 2744 problems given as daily drills. (For a detailed listing of these problems, see the Appendix.)

Table 2 presents a more detailed picture of the mean number of problems completed and the mean error rate for the various sections of the *Sets and Numbers* text material of the four books used by the children. One student started working in *Sets and Numbers*, Book 5A, but he did not complete the first section. The standard deviations given in Tables 1 and 2 reflect strikingly large individual differences;

TABLE 2. MEAN NUMBER OF PROBLEMS COMPLETED AND ERROR RATES FOR VARIOUS SECTIONS IN THE *Sets and Numbers* TEXT MATERIAL

Book Section	Mean No. of Problems Completed	Standard Deviation	Mean % of Problems in Error	Standard Deviation	N
Book 3A					
1. Sets	.....	....	....	....	..
2. Multiplication	.....	....	....	....	..
3. Carrying and Borrowing	132.5	34.6	11.2	10.4	2
4. Review	295.0	0.0	5.9	6.5	2
5. Division and Multiplication	286.0	0.0	3.8	3.0	2
6. Fractions	145.0	0.0	2.4	3.4	2
7. Subsets and Less Than	109.5	12.0	5.8	5.8	2
8. Geometry	100.0	0.0	1.5	2.1	2
Book 3B					
1. Thousands, Hundreds, Tens and Ones	152.0	134.5	10.5	5.7	7
2. Geometry	58.1	15.5	14.2	9.0	7
3. More about Sets	189.0	0.0	11.8	9.3	7
4. Commutative Laws	215.9	39.1	3.0	1.6	8
5. Associative Laws	442.3	152.7	6.6	3.7	7
6. Shapes and Sizes	65.0	0.0	4.4	7.8	6
7. Distributive Law for Multiplication	378.9	56.9	10.2	5.7	10
8. Lines of Symmetry	122.0	0.0	1.9	2.5	9
9. Word Problems	130.8	18.7	14.2	9.3	9
Book 4A					
1. Review of Sets, Addition and Subtraction	205.2	36.5	4.5	3.2	12
2. Review of Geometry	44.0	0.0	15.9	7.8	10
3. Addition and Subtraction	366.4	160.8	8.5	6.8	17
4. Geometry	99.8	9.2	15.4	8.6	18
5. Review of Multiplication and Division	309.0	0.0	2.6	2.0	17
6. Laws of Arithmetic	832.7	107.9	3.1	1.9	18
7. Shapes and Sizes	47.0	4.1	8.6	7.3	17
8. Distributive Law for Multiplication	718.5	302.3	4.9	4.5	18
9. Applications	366.7	1.0	7.2	3.8	14
10. Distributive Law for Division	499.9	113.8	4.2	2.7	14
Book 4B					
1. Circles and Lines	89.8	0.7	2.8	2.2	8
2. Subsets and Less Than	62.0	0.0	5.0	5.1	9
3. Fractions	336.1	73.6	3.2	1.1	9
4. Division	1017.0	0.0	2.5	0.0	1
5. Average	55.3	11.4	11.5	5.8	6
6. The Number Line	224.4	118.6	8.2	8.1	8
7. More about Geometry	107.7	32.8	1.7	2.0	6
8. Review of the Commutative, Associative, and Distributive Laws	521.6	25.3	3.9	2.6	5
9. Word Problems	146.0	0.0	11.6	7.2	5
10. More about Sets	179.6	13.1	10.4	10.8	5
11. Logic	112.0	19.3	0.0	0.0	4

however, many students completed only partial sections of the material, and then were encouraged by the teacher to go on to the next section.

Separation in the *Sets and Numbers* curriculum between the top and bottom student was approximately 1-1/2 years. The difference between this separation and that of the 1-3/4 years' separation for 1963-64 may be explained in part by the increasingly difficult problems presented in the advanced texts where the faster students were working. However, one of the significant conclusions emerging from the present project seems to be that the large cumulative differences observed in 1963-64, the first year of this study and the first grade for the students, are not continuing to accumulate each year. Otherwise, on a conservative basis we would expect the fastest and the slowest students in this initially highly homogeneous group to be separated by at least 3 years at the end of the third year. Further longitudinal evidence on this point with a larger sample of bright children is very much to be desired. As indicated in last year's report, the great emphasis on arithmetical skills from the fourth to the sixth grade and the relative difficulty even bright children have in perfecting these skills seems to be at least one important factor in keeping the spread from increasing.

Daily Drills

Figures 1-3 present the mean percentage of correct exercises in blocks of five consecutive drills. These figures show the consistently high performance of the students; more detailed information about each drill will be found in the Appendix. When the students found a new type of drill difficult, they showed great improvement when they next attempted a similar drill. The teachers were satisfied with the results of these short drills since most of the drills were prepared for students of at least the fourth-grade level in mathematics.

Tests

When the students completed certain sections in the *Sets and Numbers* texts, the teachers administered achievement tests on the recently completed material.

FIG. 1. Mean percentage of correct exercises in blocks of five consecutive drills, for Drills 1-55.

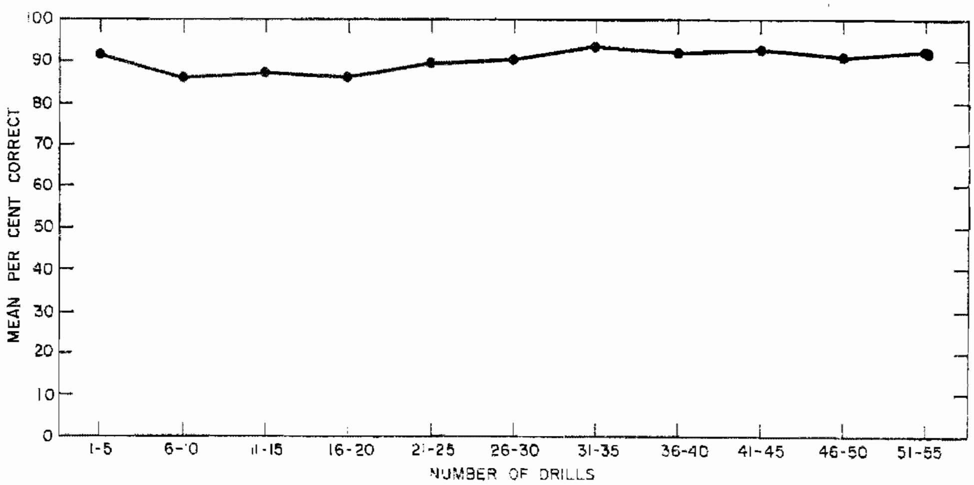


FIG. 2. Mean percentage of correct exercises in blocks of five consecutive drills, for Drills 56-110.

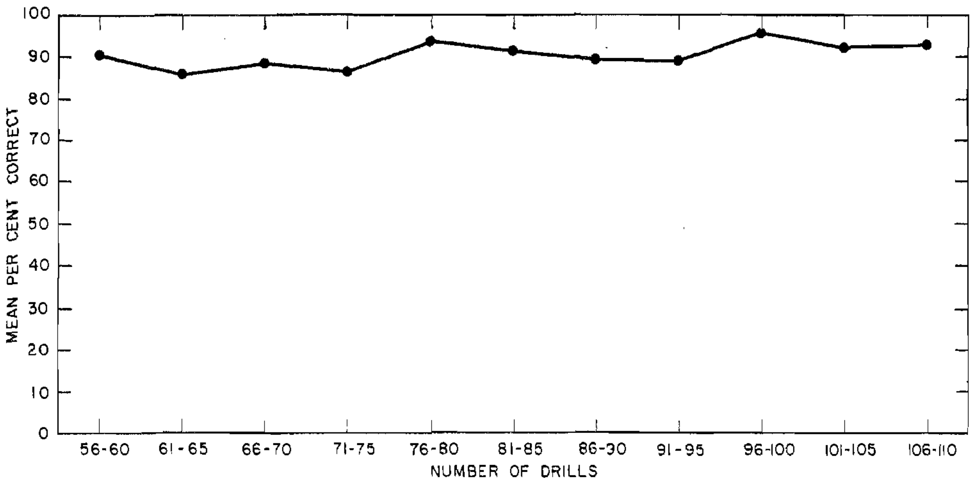
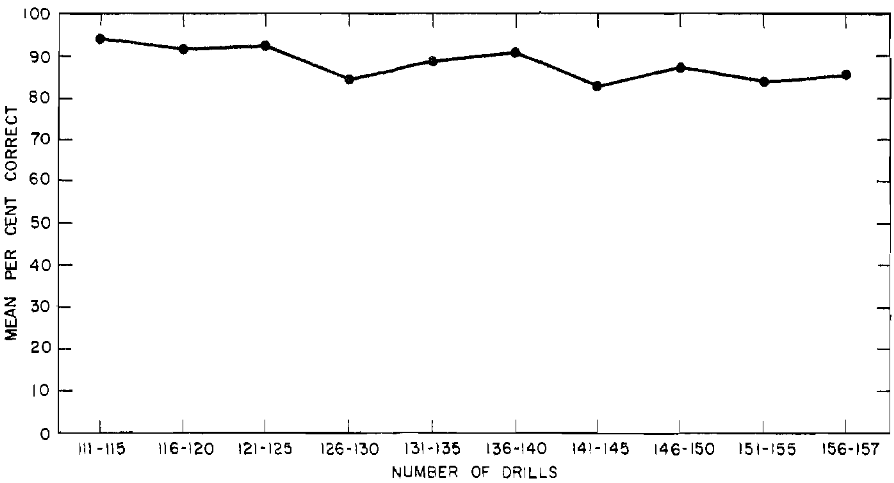


FIG. 3. Mean percentage of correct exercises in blocks of five consecutive drills, for Drills 111-157.



The results of these tests are presented in Table 3. Certain sections show a slightly higher individual variation than do the tests reported last year.

The *Greater Cleveland Mathematics Tests for Grade 3* were administered quarterly this year. Again, a slightly higher variation appeared than last year. Some of the concepts tested had not been covered by the students in their class work. At this time no normative data are available for these tests.

In the fall of 1965 the SRA (Science Research Associates) Achievement Test for Grade 3 was administered. The mean results of the test may be expressed in percentiles according to national norms: Arithmetic Concepts—96.4, Arithmetic Reasoning—95.2, and Computation—98.1. The percentile rank was for the first semester of third grade.

TABLE 3. MATHEMATICS ACHIEVEMENT TESTS

Test	Total Possible Score	Group Mean	Standard Deviation	N
Sets and Numbers				
Book 3A, Part 1	44	40.5	2.1	2
Book 3A, Part 2	40	28.8	6.7	4
Book 3B, Part 1	39	29.0	4.9	14
Book 3B, Part 2	40	34.2	3.4	18
Book 4A, Part 1	55	43.1	3.3	19
Book 4A, Part 2	20	18.4	1.4	17
Book 4A, Part 3	40	31.4	4.2	18
Book 4A, Part 4	32	29.2	2.4	17
Book 4A, Part 5	43	39.7	2.2	18
Book 4A, Part 6	30	25.2	2.9	13
Book 4A, Part 7	57	47.3	6.5	12
Book 4A, Part 8	37	33.8	2.9	11
Book 4B, Part 1	28	24.9	3.1	11
Book 4B, Part 2	68	54.9	7.7	8
Book 4B, Part 3	13	10.3	1.0	8
Book 4B, Part 4	71	64.0	13.4	6
Book 4B, Part 5	40	27.3	10.7	4
Book 4B, Part 6	53	38.8	15.4	4
Book 4B, Part 7	38	37.0	2.0	4
Book 4B, Part 8	67	50.0	0.0	1
Book 4B, Part 9	45	45.0	0.0	2
Greater Cleveland Mathematics Tests for Grade 3				
1	33	29.8	2.8	32
2	33	28.2	3.3	32
3	31	25.7	3.3	32
4	31	23.9	3.9	32

Study of the personality and social development of the students was continued under the direction of Professor Pauline S. Sears, and the results will be published elsewhere. However, there is one recent result of their work that is directly pertinent to the test results reported here. In Sears, Katz and Soderstrum (1966), the selection of a comparison group a year younger from the same four schools and with comparable ability is reported (the ability being measured by individual intelligence tests and the same group test, the New York Test of Arithmetical Meanings, used by Suppes and Hansen (1965) in selecting the accelerated mathematics group). At the end of the second grade this comparison group was given the same Greater Cleveland Mathematics Tests, Grade 2, Parts 3 and 4. The accelerated mathematics group did significantly better on these two tests (significant at the .001 level). This result would indicate that the accelerated group is not falling behind in the regular mathematics curriculum, even though a good deal of their mathematics-curriculum time is spent on special enrichment topics.

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APPENDIX

Data on Daily Drills 1965-66

Drill Number	Description	Possible Score	Mean % Correct	Number of Students
1	Sums 35-100; a + b = c; vertical format	30	91.6	32
2	Sums 35-103; a + b = c; vertical format	30	93.7	31
3	Sums 55-1100; a + b = c; vertical format	30	87.8	32
4	Sums 21-93; a + b + c + d = e; vertical format	20	88.3	32
5	Differences 4-9; a − b = c; vertical format	15	95.5	31
6	Differences 3-9; a − b = c; vertical format	15	90.4	32
7	Mixed addition, subtraction; sums to 25; vertical format	15	90.5	30
8	Mixed addition, subtraction; sums to 66; vertical format	15	81.1	26
9	Mixed addition, subtraction; sums to 141; vertical format	15	82.8	31
10	Mixed addition, subtraction; sums to 131; vertical format	15	86.7	29
11	Mixed addition, subtraction; sums to 276; vertical format	15	85.8	30
12	Mixed addition, subtraction; sums to 141; vertical format	15	90.1	31
13	Mixed addition, subtraction; sums to 180; vertical format	15	91.1	30
14	Mixed addition, subtraction; sums to 152; vertical format	15	90.3	29
15	Mixed addition, subtraction; sums to 340; vertical format	15	82.1	31
16	Mixed addition, subtraction; sums to 655; vertical format	12	83.0	27
17	Mixed addition, subtraction; sums to 20, 465; vertical format	12	85.0	31
18	Mixed addition, subtraction; sums to 1650; vertical format	12	87.3	32
19	Mixed addition, subtraction; sums to 1554; vertical format	12	83.6	31
20	Mixed addition, subtraction; sums to 419; vertical format	12	91.2	31
21	Mixed addition, subtraction, multiplication; vertical format	12	94.5	32
22	Mixed addition, subtraction; sums to 1401; vertical format	12	89.6	32
23	Mixed addition, subtraction; sums to 1133; vertical format	12	88.0	32
24	Mixed addition, subtraction, multiplication; vertical format	12	93.0	32
25	Mixed addition, subtraction, multiplication; vertical format	12	83.8	32
26	Mixed addition, subtraction, multiplication; vertical format	12	86.7	30
27	Mixed addition, subtraction, multiplication; vertical format	12	92.8	31
28	Mixed addition, subtraction, multiplication; vertical format	12	93.5	32
29	Mixed addition, subtraction, multiplication; vertical format	12	87.8	32
30	Mixed addition, subtraction, multiplication; vertical format	12	90.3	31
31	Mixed addition, subtraction; sums to 18; vertical format	25	98.8	31
32	Mixed addition, subtraction; sums to 14; vertical format	25	96.8	30
33	Mixed addition, subtraction, multiplication; vertical format	15	90.7	30
34	Mixed addition, subtraction, multiplication; vertical format	12	89.3	31

35	Mixed addition, subtraction, multiplication; vertical format	12	93.8	31
36	Mixed addition, subtraction, multiplication; vertical format	12	90.6	30
37	Mixed addition, subtraction, multiplication; vertical format	25	99.0	32
38	Mixed addition, subtraction, multiplication; vertical format	12	93.3	32
39	Mixed addition, subtraction, multiplication; vertical format	12	90.6	31
40	Mixed addition, subtraction, multiplication; vertical format	12	87.3	31
41	Differences 1-10; $a - b = c$; horizontal format	20	97.9	31
42	Differences 11-30; $a - b = c - d$; horizontal format	20	79.1	32
43	Sums 46-50; $a + b = c$; horizontal format	20	93.9	31
44	Multiplication by 3; $a \times b = c$; horizontal format	20	94.7	30
45	Differences 16-20; $a - b = c$; horizontal format	20	98.0	30
46	Differences 11-20; $a - b = c$; horizontal format	20	91.7	30
47	Differences 21-30; $a - b = c$; horizontal format	20	93.6	28
48	Sums 31-40; $a + b = c$; horizontal format	20	96.5	30
49	Differences 11-30; $a - b = c - d$; horizontal format	20	78.9	31
50	Multiplication by 4; $a \times b = c$; horizontal format	20	93.2	30
51	Multiplication by 5; $a \times b = c$; horizontal format	20	97.6	27
52	Differences 21-30; $a - b = c$; horizontal format	20	90.5	31
53	Differences 11-30; $a - b = c - d$; horizontal format	20	88.2	30
54	Differences 31-40; $a - b = c$; horizontal format	20	94.8	25
55	Multiplication by 6; $a \times b = c$; horizontal format	20	91.9	29
56	Differences 21-30; $(a\text{-}b) - c = d - e$; horizontal format	20	87.0	28
57	Differences 11-20; $a - b = c$; horizontal format	20	91.4	29
58	Sums 61-70; $a + b = c$; horizontal format	20	98.2	30
59	Differences 21-30; $(a - b) - c = d - e$; horizontal format	20	89.0	30
60	Mixed drill; all operations	20	88.0	32
61	Mixed drill; all operations	20	90.8	32
62	Mixed drill; all operations	20	89.3	27
63	Word problems; all operations	8	82.1	28
64	Word problems; all operations	8	76.3	31
65	Mixed drill; all operations	20	91.8	31
66	Word problems, two step	7	71.9	32
67	Word problems; equations	6	89.0	32
68	Word problems; equations	6	88.0	32
69	Division; multiplication	23	96.9	31
70	Equations	24	95.3	32
71	Sums 51-60; $a + b = c + d$; horizontal format	20	86.3	32
72	Differences 31-40; $(a - b) - c = d - e$; horizontal format	20	85.5	32
73	Sums 41-50; $a + b = c + d$; horizontal format	20	82.6	31
74	Differences 21-30; $a - b = c$; horizontal format	20	86.2	31
75	Multiplication by 7; $a \times b = c$; horizontal format	20	91.0	32
76	Mixed drill; all operations	20	87.2	30

DATA ON DAILY DRILLS 1965-66 (continued)

Drill Number	Description	Possible Score	Mean % Correct	Number of Students
77	Multiplication by 9; a × b = c; horizontal format	20	90.4	30
78	Multiplication by 10; a × b = c; horizontal format	20	98.4	28
79	Differences 21-30; a − b = c; horizontal format	20	93.8	29
80	Multiplication by 3; a × b = c; horizontal format	20	97.0	30
81	Sums 51-60; a + b = c + d; horizontal format	20	83.5	30
82	Multiplication by 4; a × b = c; horizontal format	20	97.0	30
83	Mixed drill, all operations	20	85.3	31
84	Mixed drill, all operations	20	92.7	32
85	Multiplication by 5; a × b = c; horizontal format	20	97.6	31
86	Multiplication by 6; a × b = c; horizontal format	20	94.1	31
87	Word problems; all operations	8	77.0	31
88	Multiplication by 7; a × b = c; horizontal format	20	92.6	31
89	Differences 31-40; a − b = c; horizontal format	20	87.5	26
90	Multiplication by 8; a × b = c; horizontal format	20	95.2	31
91	Sums 61-70; a + b = c + d; horizontal format	20	86.7	30
92	Multiplication by 9; a × b = c; horizontal format	20	92.9	30
93	Differences 41-50; a − b = c; horizontal format	20	93.5	27
94	Multiplication by 11; a × b = c; horizontal format	20	98.1	29
95	Word problems; all operations	8	74.0	26
96	Multiplication by 3; a × b = c; horizontal format	20	98.3	26
97	Multiplication by 4; a × b = c; horizontal format	20	96.6	28
98	Multiplication by 5; a × b = c; horizontal format	20	97.2	27
99	Mixed drill; all operations	20	87.9	26
100	Multiplication by 6; a × b = c; horizontal format	20	97.1	29
101	Multiplication by 7; a × b = c; horizontal format	20	95.4	30
102	Multiplication by 8; a × b = c; horizontal format	20	95.0	30
103	Sums 41-50; (a + b) + c = d; horizontal format	20	89.8	28
104	Multiplication by 9; a × b = c; horizontal format	20	94.1	28
105	Differences 21-30; a − b = c − d; horizontal format	20	86.6	29
106	Multiplication by 10; a × b = c; horizontal format	20	99.3	28
107	Multiplication by 12; a × b = c; horizontal format	20	89.1	31
108	Differences 21-30; a − b = c − d; horizontal format	20	89.9	31
109	Multiplication by 4; c = a × b; horizontal format	20	96.6	31
110	Differences 31-40; a − b = c − d; horizontal format	20	89.9	31

111	Multiplication by 5; c = a × b; horizontal format	20	99.5	30
112	Multiplication by 6; c = a × b; horizontal format	20	93.5	30
113	Multiplication by 7; c = a × b; horizontal format	20	95.2	29
114	Mixed drill; all operations	20	92.9	30
115	Mixed drill; all operations	20	92.2	30
116	Mixed drill; all operations	20	87.3	28
117	Mixed drill; all operations	20	88.6	32
118	Multiplication by 8; c = a × b; horizontal format	20	96.0	32
119	Multiplication by 9 ; c = a × b; horizontal format	20	93.7	30
120	Word problems; all operations	8	93.1	22
121	Multiplication by 11; c = a × b; horizontal format	20	98.1	29
122	Multiplication by 12; c = a × b; horizontal format	20	93.5	32
123	Multiplication by 5; (a × b) + c = d; horizontal format	20	93.8	32
124	Multiplication by 6; (a + b) + c = d; horizontal format	20	92.0	31
125	Multiplication by 9; (a × b) + c = d; horizontal format	20	87.1	26
126	Multiplication by 10; (a × b) + c = d; horizontal format	20	77.5	29
127	Multiplication by 11; (a × b) + c = d; horizontal format	20	85.7	31
128	Word problems; all operations	8	85.0	30
129	Differences 41-50; a − b = c − d; horizontal format	20	91.4	30
130	Sums 51-60; (a + b) + c = d; horizontal format	20	81.6	29
131	Mixed drill; all operations	20	95.0	30
132	Mixed drill; all operations	20	91.8	22
133	Word problems; all operations	8	89.4	20
134	Word problems; all operations	8	73.3	21
135	Mixed drill; all operations	20	92.6	23
136	Mixed drill; all operations	20	92.3	22
137	Mixed drill; all operations	20	93.4	21
138	Mixed drill; all operations	20	88.6	21
139	Mixed drill; all operations	20	91.8	23
140	Word problems; all operations	8	87.5	22
141	Sums 61-70; (a + b) + c = d; horizontal format	20	83.0	30
142	Word problems; all operations	8	69.0	29
143	Fractions; counting	20	90.4	30
144	Fractions; comparing to 1	20	87.6	31
145	Fractions; comparing values	20	83.2	30
146	Fractions; comparing values	20	90.0	31
147	Fractions; comparing values	20	87.8	29
148	Fractions; comparing values	20	89.8	22
149	Fractions; comparing values	20	79.5	10
150	Fractions; comparing values	20	89.5	9
151	Fractions; comparing values	20	91.5	7
152	Mixed drill	4	64.8	32
153	Sums of money; vertical format	12	94.3	32
154	Differences of money; vertical format	12	86.7	20
155	Multiplication by 1 digit; vertical format	9	83.3	20
156	Sums, differences of money; vertical format	12	81.8	21
157	Multiplication by 1 digit; vertical format	12	89.6	12