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Adding Up the NEW MATH

A short course for parents on the why's and where-fore's of the new math.

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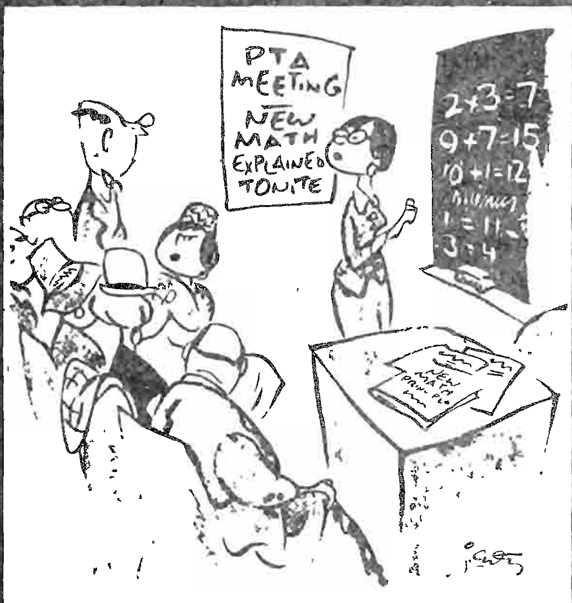
IF YOU HAVE HAD a child in elementary school at any time in the last ten years, you are undoubtedly aware of profound and perhaps puzzling changes in the math program. Chances are that you, like many other parents, have asked, "What is the point? What was wrong with the mathematics I learned when I was in school?" These questions have clear-cut answers. To begin with, it is important to realize that the changes in elementary school mathematics are not isolated and unique. During the past decade the mathematics and science curriculums in both high school and college have been subjected to broad and deep reforms. In view of these large-scale changes it would be unusual and surprising if elementary school mathematics had remained unchanged.

There are at least three good reasons for the widespread interest in changing elementary school mathematics. First, the mathematics program in the twenty or thirty years preceding these current reforms was quite limited in scope. It was almost entirely devoted to elementary arithmetic, with the accent on learning how to do simple problems of addition, subtraction, multiplication, and division. But after World War II a strong feeling developed in many quarters that elementary school mathematics was too restricted for children growing up in a highly technological age.

A second reason for the change was the substantial evidence that even the limited curriculum of the thirties and forties was very poorly mastered by students. Test results accumulated over several decades indicated that the level of accomplishment was below that which might have been expected. Attempts to explain this poor showing have led to a third reason for change.

A central criticism of the traditional math curriculum has been that it emphasized the perfection of skills rather than the understanding of concepts. When simple skills rather than understanding are stressed, a number of undesirable things can happen.

In the first place, there is good psychological evidence that skills learned in this fashion are not remembered well. For example, the child who has simply memorized the multiplication table will more easily forget multiplication facts than the child who understands how the table may be constructed, by thinking of multiplication as repeated addition. Also, the child who has learned to add, subtract, multiply, and divide with great speed, in a simple problem



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"What's all this fuss about the new math?...Why, my wife's been using it on her check stubs for years!"

format, and has not been given some understanding of these four fundamental operations, will be thrown off by even mild deviations from this format. To take the multiplication table again as an example, children who know very well that $7 \times 9 = 63$ will often have considerable difficulty in telling you what $(7 \times 9) + 3$ equals.

Some new topics

Now for a brief look at some of the topics and concepts that are stressed in the new mathematics curriculum. We may begin with geometry. Most of us are familiar with the trouble high school students have had with geometry. How, then, we may ask, could we possibly hope to introduce geometry to much younger children? The answer is that the objectives of geometry teaching in elementary school are very different from those in high school. The high school geometry course puts emphasis on teaching the student logical rigor. Traditionally this course has been his first introduction to the concept of a mathematical proof and the application of extensive logical reasoning to mathematics.

One of the difficulties in teaching geometry from a logical standpoint at the high school level has been that students have not had any intuitive acquaintance with geometry—what mathematicians call the "intuitive and perceptual basis of geometry." Yet it is easily separated from the logical analysis of geometry and is a very natural thing to teach at an early stage, before introducing children to the rather intricate problems of logical proof. Now being taught in the elementary school are such highly intuitive concepts as these: *convex and concave figures; the symmetry properties of squares, triangles, and circles, as well as the symmetry properties of familiar objects such as plates, glasses, tables, and chairs; and simple constructions with straightedge and compass, such as bisecting an angle or finding the midpoint of a line segment.*

A second example of the new topics and concepts has come out of work on the laws of arithmetic and their relation to the familiar methods of computation. Until recently many teachers—and students, too—would have a hard time explaining why we can add numbers in columns but not subtract them if there are more than two numbers. The answer is to be found in the *associative law* for the operation of addition—namely, for any numbers a, b, and c,

$$a + (b + c) = (a + b) + c.$$

On the other hand, subtraction is not associative. For example, $8 - (4 - 2) = 6$, but $(8 - 4) - 2 = 2$. Thus if we attempted to display in a column the subtraction problem $\begin{array}{r} 8 \\ 4 \\ -2 \\ \hline \end{array}$ we would get different answers, depending on whether we grouped the first and second rows or the second and third rows. This problem does not arise in addition because addition is associative. We can add up or down as we choose and always get the same answer.

The fashionable set

Another point of emphasis in the new mathematics has been the introduction of *sets*, particularly as a foundation for arithmetic. For those brought up on the traditional mathematics it may seem strange at first that this new concept should be introduced as a basis for so familiar a subject as elementary arithmetic. Yet the topic of sets, like other new topics in elementary school mathematics, is really not new. The idea of basing arithmetic on sets did not originate in the last few years; it was already well developed by the beginning of this century.

Why are sets introduced in beginning arithmetic? There are both psychological and logical reasons. Consider the psychological ones first. As we all know, the notion of a collection of things is familiar to any child. He can talk about his own toys as a collection, or even the children in his classroom. Put another way, the concept of a set or a collection of things is more concrete than the concept of a number. At the kindergarten or first-grade level children can deal with pictures of things grouped together, whereas considerable additional effort is required to use arabic numerals. The principal logical reason cannot be put quite so simply. It has to do with the mental process involved in going from concrete, visible objects to symbols. The concept of a set enables us to fill in the gaps in abstraction between physical things and arabic numerals, to which, like words in ordinary English, an arbitrary or conventional meaning has been attached.

The sets introduced to fill in this gap may be used to give a logically precise definition of number. A number is best characterized as "the number of things in a set or collection." Thus the number 2 is the number property of all collections that consist just of a pair of objects. Similarly the numerical operation of addition is that of combining collections —a general way of combining sets of things without paying attention to the things themselves. It should therefore be clear that the purpose of "set theory" in the primary grades is to provide a framework of concepts for the teaching of arithmetic itself.

Three unknown quantities

Of course it would be unrealistic to think that merely deciding what new topics might be added to the mathematics curriculum would make for an effective reform in the teaching and learning of mathematics. As is almost always the case with any educational change, ideas about how a curriculum may be modified come first. The next question is how to carry out the modifications successfully. With the new math at least three general problems have arisen, but I think we are in the process of solving them.

The first problem is that pupils in new mathematics programs have not shown much greater proficiency in the basic arithmetic skills than have children in the traditional curriculum. Not that they have done any worse, but there has been no marked improvement. It is reasonable to say that improvement might not be expected, because these new-math pupils are being taught a number of other mathematical concepts of the kind I have already mentioned. But it is equally reasonable to hope for a markedly higher level of proficiency in the basic skills. I think it is fair to say that this problem has not yet really been tackled by those working with the new math curriculum. I predict, however, that we shall see more and more concern about it in the next decade.

Problem number two: More time is needed to train teachers in the new mathematics. Take geometry, for instance. A reasonably high percentage of elementary school teachers have not had a high school course in geometry. True, many of them have mastered the basic facts of geometry needed for elementary school teaching by in-service training or summer programs. Moreover, throughout the country the mathematics requirements for certified elementary school teachers are being upgraded. So I think that in the future we may expect more thorough training of elementary school teachers.

A third problem is to make a distinction between *what is possible to teach* and *what is the most desirable to teach.* From the various experimental projects in the teaching of elementary school mathematics we know that it is possible to teach a child, whatever his ability, a good deal more math than we have done in the past. However, it will take some time and experience to decide which of the topics that *can* be taught in elementary school are also desirable at that level. Of course, the problem of distinguishing the desirable from the possible is by no means limited to the mathematics curriculum. It is a profoundly difficult one in all parts of the curriculum.

Small change

Any fundamental change in curriculum is bound to produce problems, but acceptance of change in elementary school mathematics now seems widespread. I think the reasons for this general acceptance are fairly clear. For one thing, the changes being made are relatively conservative. For example, no one has advocated neglecting the basic skills and facts of arithmetic. The new topics are not radically new in mathematics. In the case of geometry we are talking about topics that were familiar to school children in ancient Athens more than two thousand years ago.

The new mathematics curriculum is also conservative in that the new concepts do not raise to complicated social and political issues—as sometimes happens with other parts of the school curriculum. Fortunately there is no dispute among mathematicians about the truth of any of the propositions taught in the new mathematics. The only disagreement is about their usefulness or desirability in the curriculum.

Mathematics is one of the most exciting fields in the whole spectrum of the sciences. It seems right and proper that the mathematics we teach our children should now begin to have some of the depth and imagination so characteristic of modern mathematics itself.

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