Arithmetic drills and review on a computer-based teletype*

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Given a computer-based instructional program and a school in which to work, it is possible to supplement or enrich the teacher's instruction by taking over the more routine daily tasks, presenting special materials, or giving the daily lesson itself. The task of the present project was to prepare a program in arithmetic to review and teach the basic number facts as a supplement to the teacher's daily instruction at the fourth-grade level. In addition, the project was concerned with gaining a clearer picture of the optimum teacher-machine interaction pattern that would take full advantage of both.

Objectives

One of the primary objectives was to review and teach the basic number facts which comprise an important part of a fourth-grade mathematics program. Often those engaged in teaching "modern mathematics" play down this part of the curriculum. Stressing fundamental concepts and structure is essential, but mastery of the basic facts should not be neglected.

In working out a continuous daily program, the sequencing of material and the provision for a proper amount of review to correlate materials with the day-to-day classroom instruction become practical objectives. In our case both level of difficulty and length of exercise directly affected each student's running time, which had to be sufficiently short to permit each student in the class to perform on the machine during the school day.

To our knowledge no elementary school teacher has had a computer-based teaching device available to her on a daily basis up to this time. The mechanics of having each child take his turn during the day without disrupting the regular work of the class was of some concern. Related to this problem was concern over the ability of a teacher, untrained in the use of the machine or its operation, to adjust to its presence in the room and to use it optimally. From previous experience in observing young children operate the teaching machines in Stanford's Computer-Based Instruction Laboratory it was believed that there would be little or no problem in the children's adjusting to the machine. (This assumption was correct; most students were very quick to master the simple operations required.)

Related research

Programmed instruction

Reports of extensive research over long periods of time are lacking. While some creditable work has been done in the areas of branching and feedback variables, few studies using computer-based teaching devices are available. Most re-

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ported studies may be summarized by saying that students using well-written programmed materials, whether in a text or a machine-like device, will be able to master some material as well as students in a regular class in somewhat less time.

Drill in arithmetic

Some confusion still exists in the minds of many people concerning the use of drill in today's curriculum. Brownell [1]* has pointed out the need for well-planned drill in commenting on readiness for division: "If children find the topics hard, many times it is due to inadequate mastery of the skills and basic facts needed." Some years ago Buswell [3] reported finding that 93 percent of the errors in long division made by fourth-, fifth-, and sixth-grade pupils was due to a lack of mastery of number facts rather than number processes. Spencer [4] maintained that there are indeed "typical errors in arithmetic accounting for 75 percent of all the errors in addition, 80 percent of the errors in subtraction, 50 percent of the errors in multiplication, and 70 percent of the errors in division."

The dangers of teaching by drill methods alone were pointed out in a later study by Brownell and Chazel [2]. It was emphasized that effective teaching must precede drill, if the drill is to have the desired results. It seems apparent on the basis of the aforementioned studies that drills can most effectively be used to overcome the large percentage of typical errors in arithmetic after an introduction to the subject has been given. Also, a student should be given an opportunity to correct errors he makes in his daily work. Suppes and Ginsberg [5] found that young children, who overtly corrected incorrect responses, performed significantly better than a noncorrection group on a concept-formation task.

After studying a large number of reports on drill methods, Wilson [6] concluded that, to be effective, a drill should have the following attributes:

1 It should be on the entire process.
2 It should come frequently in small amounts.
3 Each unit should be a mixed drill.
4 It should have a time limit.
5 Examples in a drill should be in order of difficulty.
6 Drills should include verbal problems.
7 Drills should facilitate diagnosis.

Procedure

The machine and the classroom

The teaching machine used in this project was a commercially available teletype connected by private telephone line to the computer in Stanford's laboratory. A large book closet which opened into the classroom was modified by adding a ventilation fan, light, and electrical outlet. This provided privacy for the user, and insulated the rest of the class from the operational noise of the teletype. With these very minor modifications the closet provided an excellent teaching station throughout the day.

Instructional program

Instruction on the machine began in the spring of 1965 and ran for seven consecutive weeks. The daily drills were based on the principles cited in the above-mentioned research, particularly in terms of the attributes listed by Wilson. Each drill was short, three to six minutes, varying from five to thirty problems (with an average of twenty).

As each student took his turn, the machine printed out "please type your name." The student spelled his name by typing, using the hunt-and-peck method for the most part. If his name was incorrectly spelled, he was informed, "This name is not on the student list, try again. Please type your name." A proper entry set the program in operation and the first problem was printed out, leaving a blank for the correct response. The machine was programmed to position itself at the blank so as to have the response

* Numbers in brackets refer to the References at the end of this article.

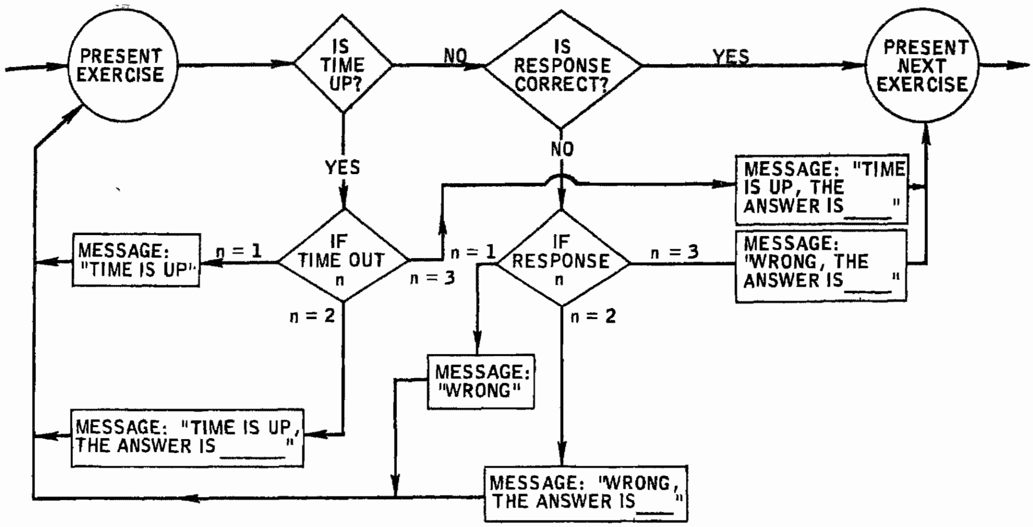

Figure 1. *Flow chart of the program logic for computer-based instruction in the fundamentals of arithmetic.*

properly placed. A correct response was reinforced by the appearance of the next problem. An incorrect response was indicated by the typed-out word "wrong," and the problem itself was repeated. A second error on the same problem was followed by the message "wrong, the answer is ______," and the correct answer was displayed. The problem itself was then given once more to allow for a correction response. An error on the correction response would cause the previous message to reappear. The next problem would then be presented. A ten-second time limit per response was set. If a response was not given in ten seconds, the machine response followed the above pattern, except that the words "time is up" were substituted for "wrong" at each step. The sequence of events is presented in Figure 1.

When the lesson was completed, the machine printed out the following information for the student: total errors (number of errors), problems missed (the number of each problem on which an error or time-out was made), and total elapsed time in seconds. Following this information, it typed "tear off here _ _ _ _ _ _" and turned the paper up to the cutter bar, permitting the student to tear off and keep the printed record of his day's work.

At the end of the day or when all the children had finished, the teacher typed the word "finished." The computer program then gave her (1) the number of students who made time-outs and errors on each problem in the lesson; (2) the distribution of error for the entire class, i.e., the number of students making 0, 1, 2, etc., errors and time-outs, and (3) the distribution of the total elapsed time by thirty-second intervals for the class.

Teacher preparation

Little teacher preparation was required. A simple dial-in code was all that was needed to call up each day's lesson. The code, consisting of ten steps, was posted on the machine for the teacher to follow.

The class

The class consisted of 41 fourth-grade pupils. Their average IQ on the CTMM was 122. They adapted very quickly to the machine and had few, if any, problems after the second day with either operating the machine or finding the right key.

Supervision

One project staff member was either present or on call by phone at all times. Constant attention was required at the beginning of the experiment. However, the number of breaking-in problems of the new operation soon diminished, and after the first three weeks the teacher controlled the daily operation alone and without difficulty.

Lesson content

The data presented in this paper are based on fifteen lessons. Prior to these fifteen lessons, each student had been given two practice lessons to make him familiar with the equipment.

Lesson 1 contained problems of the form $(6\times7)+3=$____, $(57-3)\div6=$____, and three problems of the form $53-4=7\times$____. Lesson 2 concentrated on problems of the form $(8\times6)-6=6\times$____ and $(7\times5)+5=4\times$____. Lesson 3 was on units of measure containing items such as 1 qt. = ____ pt., 1 mile = ____ ft., and 2 yds. + 5 ft. = 3 yds. and ____ ft. Lesson 4 contained such problems as $(4\times3)\times3=$____, $147\times4{,}352=4{,}352\times$____, and $3\times(4+7)=(3\times4)+(3\times$____$)$.

Beginning with Lesson 5, the problems were of a simpler form. Lessons 5, 6, and 7 were on the multiplication tables for 8, 9, and 10. Each problem had the general form $a\times b=$____. Lesson 8 was a mixed drill using the operations of addition, subtraction, multiplication, and division. Lessons 9 and 10 concentrated on multiplication by 10 and 100. Lesson 11 consisted of five word problems. This was the first time the children had been exposed to word problems on the teletype. One of the problems asked students to find distance, given rate and time; two problems required simple mental division; one was a simple subtraction problem involving money, and the fifth was an addition problem concerned with tickets to a school play. Lessons 12, 13, and 14 were mixed drills containing a large proportion of simple multiplication problems. Lesson 15 was another drill on units of measurement.

Findings

The data obtained from this procedure were summarized on the basis of the first answer given by the pupil. If his answer was wrong or timed-out, it was regarded as such, regardless of the response he made on his second attempt at the problem. Our main reason for distinguishing between errors and time-outs was that if a pupil was timed-out, it was still uncertain whether or not he would have subsequently made an error had he been given more time.

For each lesson, the average proportions of errors, successes, and time-outs were computed. The result of this computation is shown in Figure 2. Since the drills varied

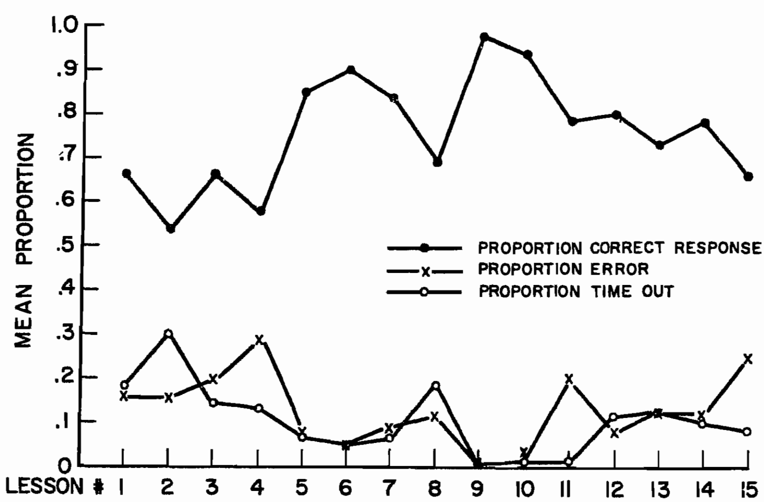

Figure 2. Probability of response per pupil on fifteen lessons.

Table 1
Rank order of lessons according to mean proportion of successes

Rank	*Lesson*	*Type*	*Predominant form*	*Number of subjects*	*Proportion of successes*
1	2	Mixed	$(a \circ b) \circ' c = d \times$ __	22	.54
2	4	Mixed	$(a \circ b) \circ' c =$ __	39	.58
3	1	Mixed	$(a \circ b) \circ' c =$ __	23	.66
3	3	Units of measure		19	.66
3	15	Units of measure		39	.66
6	8	Mixed		36	.69
7	13	Mixed	$a \times b =$ __	33	.73
8	11	Word		39	.78
8	14	Mixed		38	.78
10	12	Mixed	$a \times b =$ __	36	.80
11	7	Multiplication	$a \times b =$ __, $a = 8, 9, 10$	39	.84
12	5	Multiplication	$a \times b =$ __, $a = 8, 9, 10$	40	.85
13	6	Multiplication	$a \times b =$ __, $a = 10, 100$	24	.90
14	10	Multiplication	$a \times b =$ __, $a = 10, 100$	39	.94
15	9	Multiplication	$a \times b =$ __, $a = 10, 100$	39	.98

considerably in the type of problem that predominated, Figure 2 is hard to interpret. About all that can be said is that there is a tendency for the proportion of successes to increase, although there are many obvious exceptions. Because of this somewhat serious confounding between time and lesson type, a more meaningful comparison was made by rank-ordering the lessons in terms of proportion of correct responses and by classifying them in terms of predominant type of problem. The result of this analysis is shown in Table 1. In this table the predominant form is defined as the form of at least three-quarters of the problems in a given lesson. The symbols $\circ$ and $\circ'$ denote arbitrary but distinct operations.

It would appear from the results in Table 1 that the difficulty of a lesson is related to the type and predominant form of the problems in the lesson. The extent to which the difficulty of a lesson is related to the amount of previous practice is impossible to determine. It should be noted that, in addition to the problem of differing lesson types, any sequential analysis is certain to be confounded by the fact that the number of pupils who took each lesson varied. This can be seen by comparing Lessons 3 and 15. The two are of the same type, but do not differ in difficulty (as measured by proportion of correct responses), despite the fact that one was presented on the third day and the other on the fifteenth day. However, any inference that Lesson 3 did not facilitate Lesson 15 is made impossible by the fact that twenty students who tried Lesson 15 did not try Lesson 3.

The extent to which the proportion of correct responses is related to lesson type can be seen in the fact that lessons of the same type tend to be grouped in adjacent ranks. Thus, the five easiest lessons are the five multiplication lessons. The two lessons on units of measurement both rank 3. Moreover, the two lessons involving multiplication with $a = 8$ or 9 are adjacent in their rank-ordering, as are the three lessons involving multiplication with $a = 10$ or 100.

Figure 3 summarizes another interesting finding of the project. The graph of the mean proportion of errors roughly parallels the graph of the mean time taken to complete each lesson. It should be recalled that a time-out is not counted as an error. The relationship between time and errors appears to be correlated over the various types of lessons: number facts, units of measure, word problems, and the more complex problems. The rank-order correlation between the average proportion of errors and the time taken to complete a

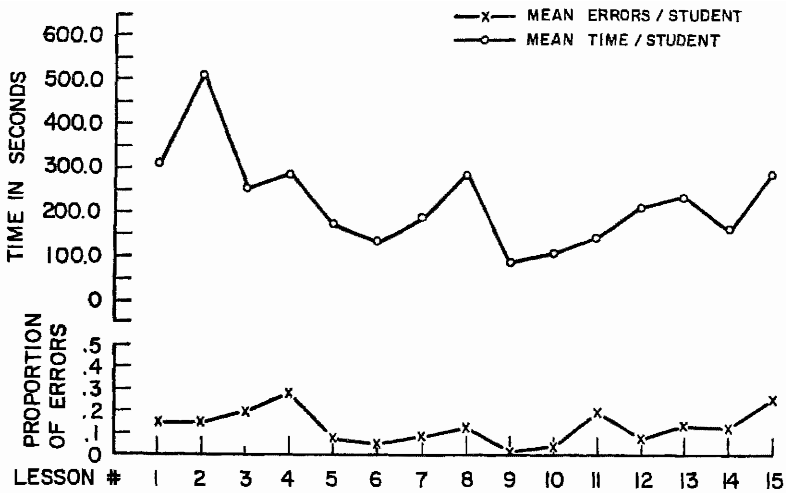

Figure 3. Mean completion time and mean proportion of errors per lesson.

lesson is 0.65, which is significant at the .05 level.

Table 2 may be used to compare individual differences between five selected students. Student 3 was promoted from second to fourth grade. Students 1 and 2 had previously demonstrated high ability and performance. Students 4 and 5 were average in ability and performance. The table generally shows the same relationship between proportion of errors and total time.

Feasibility of the system

Teacher reaction

Although only one classroom teacher was directly involved in this project, her reaction to this project and the machine was very encouraging. She remained enthusiastic throughout the project. For example, she made it possible to extend the running time through the morning recess or lunch hour on days when drills were difficult so that every student could participate. She discussed with individual students the errors on their print-outs, and sometimes modified her instruction to handle problems of general concern to the class.

Limitations imposed by the system on the class and curriculum

Time was an important factor in daily operations during the project. The originally planned four-minute allotment for each student was difficult to maintain. Frequently, students would run over the planned time, causing the day's run to

Table 2
Individual differences among selected students

Pupil	*Total time to complete drill (in seconds)*			*Mean proportion wrong*	*Mean proportion correct*
	Minimum	*Maximum*	*Average*		
1	95	233	135	.06	.92
2	87	263	161	.07	.89
3	101	358	194	.11	.83
4	95	747	328	.15	.57
5	103	802	348	.15	.52

extend through the lunch hour and into the afternoon. Length and difficulty of each lesson were the major factors. Delay between students increased total running time in some cases. Generally, however, there was little delay between students due to good management by the teacher.

The lesson material itself was limited to the characters available on a standard typewriter. However, this was not a severely limiting factor, due to the type of material used in the project. Special print wheels with more mathematical symbols will be available for next year and will greatly increase the flexibility of the system. The material was presented in linear form, requiring every student to do the same lesson each day. The possibility of branching to either more or less difficult material based on performance was not available. This feature will be part of next year's program.

System operation

The major source of system failures or operational delays was in Stanford's laboratory itself. Failure to maintain priority to access memory in storage accounted for several three- to five-minute delays. Computer-component failures accounted for 80 percent of the time lost. One day was lost, due to a telephone-line transmission problem. Reliability is expected to improve as the system completes its "breaking-in" phase.

Possibilities for controlled classroom experimentation

A system, such as the one used in the project, provides an opportunity for controlled classroom experimentation. It resembles a psychological laboratory in that it provides the possibility of establishing strict control over many variables. The data reported in the present report represent a very superficial beginning and are intended only to give a sense of the methods and procedures that may be used for extensive pedagogical and psychological investigation of arithmetic skills.

Future curriculum plans

The project will be expanded during 1965–66. Three teletypes on a full-time basis will be used, one each in Grades 4, 5, and 6. Arrangements are being made at the school to allow easy access, for 80 students a day, to each machine. Data analysis and lesson programs are being improved to provide more extensive reports to students and teachers, as well as to provide branching capabilities based on student performance. Each student may be given one of five lessons each day depending on his performance on the previous day. The five possible lessons vary widely in difficulty to provide a "floor" for the poor learners and a very high "ceiling" for the fast learners. The complete program and lesson assignment will be automatic. All necessary information will be held in computer storage.

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