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12

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Four Programs in Computer-assisted Instruction[1]

Four of the major programs in computer-assisted instruction developed in the Institute for Mathematical Studies in the Social Sciences at Stanford University will be presented. These programs provide examples of the various methods of integrating a computer system into an educational setting, the specific role the computerized aspect of education can assume, and the range of curriculum taught by such a system.

Drill-and-Practice Program

The most extensive program is a drill-and-practice program in arithmetic that involved, by the end of the 1966–1967 academic year, the participation of more than 1500 students in grades 1 through 6. Although the drill-and-practice program was the most extensive in terms of students and sheer amount of curriculum material, the role of the program in the educational setting was supplementary; that is, the program was designed to drill and review students on concepts previously presented in the classroom

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by the teacher. Therefore a close temporal relationship between the learning of a concept and drill on that concept was not part of the program design. In fact, the temporal relationship between the material in the program and the mathematics curriculum in the classroom at any given point in time was determined by each teacher, not by our research staff.

For the 1966–1967 academic year the curriculum material, for each of grades 1 through 6, was arranged sequentially in blocks to coincide with the development of mathematical concepts introduced in several text series. There were 20–27 concept blocks for each grade level. Each concept block included a pretest, five days of drill, a posttest, and sets of review drills and review posttests. A brief description of the material in each concept block is shown in Table 12–1.

Table 12–1. Concept Blocks for Grades 1–6—Drill-and-Practice Program 1966–1967

Grade 1		Grade 2	
Block	**Description**	**Block**	**Description**
1.	Counting, how many, 0–9	1.	Addition facts to 10, horizontal
2.	Counting in sequence	2.	Subtraction facts to 10, horizontal
3.	Sums to 4	3.	Addition and subtraction facts to 10, vertical
4.	Sums to 4, vertical, mixed	4.	Addition facts to 10, mixed horizontal and vertical with variables
5.	Differences to 4, vertical, mixed	5.	Mixed addition and subtraction to 10, mixed horizontal and vertical
6.	Sums to 6, vertical, mixed	6.	Counting by 1's and 2's, finding what comes before and after
7.	Sums to 7, vertical, mixed	7.	Addition, 11, 12, 13; horizontal and vertical
8.	Differences to 7, vertical, mixed	8.	Subtraction, 11, 12, 13; horizontal and vertical
9.	Sums to 9, vertical, mixed	9.	Mixed addition and subtraction, horizontal and vertical to 13
10.	Sums to 10, vertical only	10.	Units of measure, counting, inequalities
11.	Differences to 10, vertical only	11.	Addition, 14, 15, 16; horizontal and vertical
12.	Sums to 10 with variables	12.	Subtraction, 14, 15, 16; horizontal and vertical
13.	Differences to 10 with variables	13.	Mixed addition and subtraction, 14, 15, 16; horizontal and vertical
14.	Sums and differences to 10, horizontal	14.	Word problems, units of measure, counting to 200
15.	Sums and differences to 10, vertical format	15.	Fractions, ½, ⅓, ¼
16.	Sums and differences to 10 with variables	16.	Addition, 17, 18, 19; horizontal and vertical
17.	Sums to 10, 3-digit numbers	17.	Subtraction, 17, 18, 19; horizontal and vertical
18.	Column addition, sums with 10's, no regrouping	18.	Mixed addition and subtraction, horizontal and vertical
19.	Column subtraction, no regrouping	19.	Units of measure, counting, inequalities
20.	Mixed addition and subtraction in columns, facts to 10		
21.	Mixed addition and subtraction, inequalities		
22.	Mixed 1- and 2-digit column addition and subtraction		
23.	Sums to 10, form $a + b = c + d$		
24.	Sums to 10 with variables, form $a + b = c + d$		
25.	Special addition and subtraction		
27.	Special mixed drills		

Grade 2 (cont.)	
Block	Description
20.	Multiplication, 2's and 3's to 9 (i.e., 2 × 0 = __ . . . 2 × 9 = __)
21.	C and A laws for addition, subtraction, multiplication
22.	Achievement tests
23.	Mixed drill: fractions, units of measure, inequalities, multiplication
24.	Mixed drill: addition, subtraction, multiplication
25.	Special addition and subtraction
27.	Special mixed drills

Grade 3		Grade 4	
Block	Description	Block	Description
1.	Mixed addition and subtraction, horizontal format, sums 0–18	1.	Addition, 1 and 2 digit, vertical and horizontal
2.	Addition, sums 0–18, horizontal and vertical	2.	Subtraction, 1 and 2 digit, vertical and horizontal
3.	Subtraction, sums 0–18, horizontal and vertical	3.	Subtraction, 2 and 3 digit, vertical format
4.	Addition, no carry, vertical (2 addends, 3 digit) and (3 addends, 2 digit)	4.	Addition, 2 and 3 digit, column addition
5.	Subtraction, no borrow, vertical, 2 and 3 digit	5.	Mixed addition and subtraction, vertical format, limits same as blocks 3 and 4, word problems
6.	Addition, vertical with carry	6.	Measure: length, time, weight, money; some word problems
7.	Subtraction, with borrow	7.	Multiplication, 2's → 9's, horizontal format, levels by products
8.	Mixed addition and subtraction, carry and borrow	8.	Mixed addition, subtraction, and multiplication; addition and subtraction, vertical format; multiplication, horizontal format; limits same as blocks 3, 4, 7; word problems
9.	Measure and word problems and inequalities	9.	CAD laws: days 1–4 apply law, day 5 identify law
10.	Column addition and subtraction, addition, subtraction	10.	Division: ladder form, no remainders, level by products, single-digit divisor, 2's → 9's
11.	Measure, inequalities	11.	Multiplication: 2's through 12's, horizontal format, level by products
12.	Multiplication, horizontal, 2's and 3's	12.	Fractions: identify (to ⅛), simple reducing
13.	Mixed multiplication and division, 2's and 3's	13.	Mixed drill: multiplication, division, fractions; inequalities; word problems; same limits as blocks 10, 11, 12; horizontal and vertical
14.	Division, ladder form, 1 digit into 2 digit		
15.	CAD laws: addition, subtraction, multiplication		
16.	Mixed drill: measure, word problems, inequalities		
17.	Fractions		
18.	Multiplication, horizontal, 2's → 9's		
19.	Mixed drill: multiplication, division, fractions		

Grade 3 (cont.)	
Block	**Description**
20.	Division, ladder form, 1 digit into 3 digit
21.	Multiplication, vertical, 1 × 2 digit
22.	Achievement tests
23.	Mixed drill: column addition, subtraction, multiplication
24.	CAD laws
25.	Special addition and subtraction drills
27.	Special mixed drills

Grade 4 (cont.)	
Block	**Description**
14.	Long division: ladder form, 1-digit divisor, 2–4-digit dividend, random divisors
15.	CAD laws: days 1–3 use, days 4–5 identify
16.	Fractions: addition, subtraction, reducing
17.	Measure: time, money, liquid measure, length, weight; some word problems
18.	Multiplication: multiples of 10, inequalities
19.	Mixed drill: multiplication; division; fractions; CAD laws; same limits as blocks 14, 15, 16, 18; some word problems
20.	Long division: ladder form, 1-digit divisor, 2–4 digit dividend, random remainders
21.	Fractions
22.	Achievement tests
23.	Mixed drill: long division; fractions; negative numbers; same limits as blocks 20, 21, 22
24.	Estimation of quotients in division
25.	Special addition and multiplication drills
26.	Special subtraction and division drills
27.	Special mixed drills

Grade 5	
Block	**Description**
1.	Addition; vertical and horizontal; 1, 2, 3 digit; level 4, carry to 10's; level 5, carry to 10's or 100's
2.	Subtraction, vertical and horizontal, 1 and 2 digit
3.	Mixed addition and subtraction, 3 and 4 digit, mixed borrow, carry
4.	Multiplication, 2's → 12's, level by products, horizontal
5.	Multiplication, vertical, up to 1 × 4 digit, carry, no carry
6.	Mixed drill: multiplication, division, fractions
7.	Division, ladder form, level 3: 1 into 3 digit
8.	Measure

Grade 6	
Block	**Description**
1.	Mixed drill: ½ column, addition, subtraction, ½ multiplication, some involving decimals
2.	Multiplication: 2's → 12's, level by products, horizontal format
3.	Column multiplication: (1 digit) × (2 digit) through (2 digit) × (3 digit)
4.	Division: ladder form, 1-digit divisor
5.	Fractions: factors, reducing, comparing, simple addition, subtraction
6.	Mixed drill: inequalities, decimals, word problems, exponents, addition, subtraction, multiplication, division
7.	Division: ladder form to 2-digit divisors

Grade 5 (cont.)		Grade 6 (cont.)	
Block	Description	Block	Description
9.	Multiplication, vertical, 2 digit, 2's → 12's	8.	Fractions: addition, subtraction
10.	Mixed drill: column addition, subtraction, multiplication, decimals, CAD laws	9.	Measure: length, time, money, temperature, liquid measure
11.	Division, ladder format, level 3: 2 into 3 or 4 digit	10.	Ratio: per cent
12.	Fractions	11.	Division: ladder form, 2-digit divisor
13.	Measure, decimals	12.	Mixed drill: fractions (addition, subtraction, multiplication), ratio, per cent, division, decimals (addition, subtraction)
14.	CAD laws	13.	Fractions: decimals, addition, subtraction, multiplication
15.	Division, ladder format	14.	CAD laws: days 1–4 apply, day 5 identify
16.	Fractions	15.	Multiplication: multiples of 10, horizontal format
17.	Mixed drill: multiplication, division, fractions	16.	Division: ladder form, 2-digit divisors, 3- to 5-digit dividends
18.	Measure	17.	Mixed drill: fractions (+, −, ×, ÷), fractions (column addition), CAD laws, division
19.	Fractions, decimals	18.	Measures: all, including a few metric, area, volume
20.	Mixed drill: multiplication, division, decimals	19.	Ratio, per cent
21.	Division, ladder format	20.	Mixed drill: all operations, per cent, decimal multiplication
22.	Achievement tests	21.	Negative numbers: addition, subtraction, multiplication
23.	Mixed drill: summary	22.	Achievement tests
24.	Estimation of quotients in division	23.	Mixed drill: summary
25.	Special addition and multiplication drills	24.	Estimation of quotients in division
26.	Special subtraction and division drills	25.	Special addition and multiplication drills
27.	Special mixed drills	26.	Special subtraction and division drills
		27.	Special mixed drills

Four parallel forms of a test, A, B, C, and D, were prepared for each concept block. The test consisted of an equal number of problems from each of five levels of difficulty. Three of these forms (A, B, and C) were used as pretests and posttests for the block, with each student randomly assigned to one of these forms as the pretest and another as the posttest. A student assigned to a given sequence of forms, for example, form A pretest and form B posttest, received the same sequence for every block. The remaining form and form D were divided into halves to be used as review posttests. For each day of drill, five drills, one at each of the five levels of difficulty, were prepared; a total of twenty-five drills per block. Several sets of review drills for each block were also prepared at the five defined levels of difficulty.

Each student was given his problems individually in the school on a computer-based control terminal connected to the PDP-1 at Stanford via telephone lines. The student responded on a Model-33 teletype with a modified keyboard. After the student signed into the program by typing his assigned student number and his first name, the teletype would print his last name and present the appropriate set of problems. The temporal pace of the problem presentation was determined by the student.

The materials presented to the student for the seven days required for each concept block were

Day 1	pretest
Days 2–5	drill and review drill
Day 6	drill and review posttest
Day 7	posttest

Examples of the format for several types of problems are shown in Figure 12–1. The teletype would print out each individual problem and then position itself to accept the answer in the appropriate place. The student would type in the answer. If his answer was correct, he would proceed to the

GRADE	BLOCK	PROBLEM
1	1	HOW MANY M'S... R M R R M M M R M M R M ...
1	2	COUNT. 10 11 ___ 13
1	4	3 + 1 = ___
2	4	9 + 1 = 5 + ___
2	5	7 + N = 9 N = ___
2	9	1 1 +__2
2	9	1 0 =__3
3	1	___ + 35 = 38
3	4	2 3 1 4 +2_1

GRADE	BLOCK	PROBLEM
4	2	3 6 :2_3
4	6	3 YD. AND 2 FT. = ___ FT.
4	9	36 X (28 + 34) = (___ X 28) + (___ X 34)
5	4	___ X 11 = 33
5	5	2 9 4 ___X4
5	6	1/3 OF 18 = ___
6	4	5 / 9 5
6	5	TYPE THE MISSING NUMERATOR OR DENOMINATOR. 2/3 = ___/9
6	6	TYPE < OR = OR > 3 + 8 ___ 9 + 4
6	7	(17 X ___) + 9 = 28722

Figure 12–1.
Samples of problem formats for grades 1 through 6, drill-and-practice program.

next problem. If he input the wrong answer, the teletype would print out NO, TRY AGAIN and present the problem again. If he made a second error, the teletype would print out NO, THE ANSWER IS . . . and present the problem once more. If the student input the wrong answer for the third time, he would be given the correct answer and the teletype would automatically proceed to the next problem. The student was allowed from ten to forty seconds to respond, depending on the type of problem presented. If a student took more than the allotted time to input his answer, the procedure just described would be followed, but the teletype would print out TIME IS UP, TRY AGAIN in place of NO, TRY AGAIN.

The level of difficulty of the first day of drill was determined by the student's performance on the pretest according to the criteria presented in Table 12–2. The level of difficulty of each successive drill in the same concept block was determined by the student's performance level on the preceding day's drill. Thus, if the student's performance on a drill was 80 per cent or greater, his next drill was one difficulty level higher. A score of less than 60 per cent would branch him down a level for the next drill. Otherwise, the student would remain at the same difficulty level for the next drill.

Whereas the content of the drill was the same for all students in a class with only the difficulty level changing as a function of the preceding day's performance, the content of the review drills differed among students as a

Table 12–2. Branching Criteria

From pretest to drill[a]		From drill to drill	
Per cent correct	Level assigned for drill	Per cent correct on drill D_i	Level assigned for drill D_{i+1}
0–19	1	0–59	Next lower level
20–39	2	60–79	Same level as D_i
40–59	3	80–100	Next higher level
60–79	4		
80–100	5		

[a] *Also from posttest to review.*

function of the total past history of each student. The computer individually selected the review drills to correspond to the content of that past block having the lowest posttest score for that student, with the restriction that he was not reviewed for two seven-day blocks in a row on the same past block. The level of difficulty of the review drills was determined by the posttest according to the criteria presented in Table 12–2; the difficulty level remained constant for all four days of review. After a student had received a set of review drills on a given concept block, the score on the review posttest given on the sixth day, replaced the previous posttest on that concept block for determining the concept block and difficulty level for future review drills.

The branching structure for a seven-day sequence of problems is shown in Figure 12–2. Each darkened circle represents a drill; each open circle represents a review drill. To make up for absences, a student could take more than one drill per day, branching accordingly after each drill.

To summarize, the basic features of the drill-and-practice program are

1. Role—drill, a supplement to the teacher's regular classroom instruction.
2. Age group—grades 1 through 6.
3. Curriculum—arithmetic.
4. Individualization:
   a. Temporal pace of problem presentation determined by the student.
   b. Immediate feedback.
   c. Opportunity for second response if first one incorrect.
   d. Level of difficulty of drills changes as a function of individual performance.
   e. Concepts for review selected for each student as a function of his past performance.
5. External materials—none.
6. Number of students—more than 1500 in 1966–1967.
7. Presentation of curriculum—teletype.
8. Response mode—standard keyboard (with some extra mathematical symbols).

Brentwood Tutorial Mathematics Program

The Brentwood program is in direct contrast to the drill-and-practice program. The system was tutorial rather than drill, teaching mathematics to forty-nine first graders during the 1966–1967 academic year. Also, the computerized aspect of learning elementary mathematics was completely integrated with the classroom work; a member of our staff taught all mathematics that was not presented in the computer program. Curriculum material was presented by audio and visual displays; the student responded on a standard keyboard or used a light pen to touch one of the answer choices displayed on the cathode-ray tube (CRT).

There were 400 lessons covering the topics of counting, numerals, addition, subtraction, linear measure, sets and set notation, and geometry. The content and scope of the curriculum were drawn largely from *Sets and*

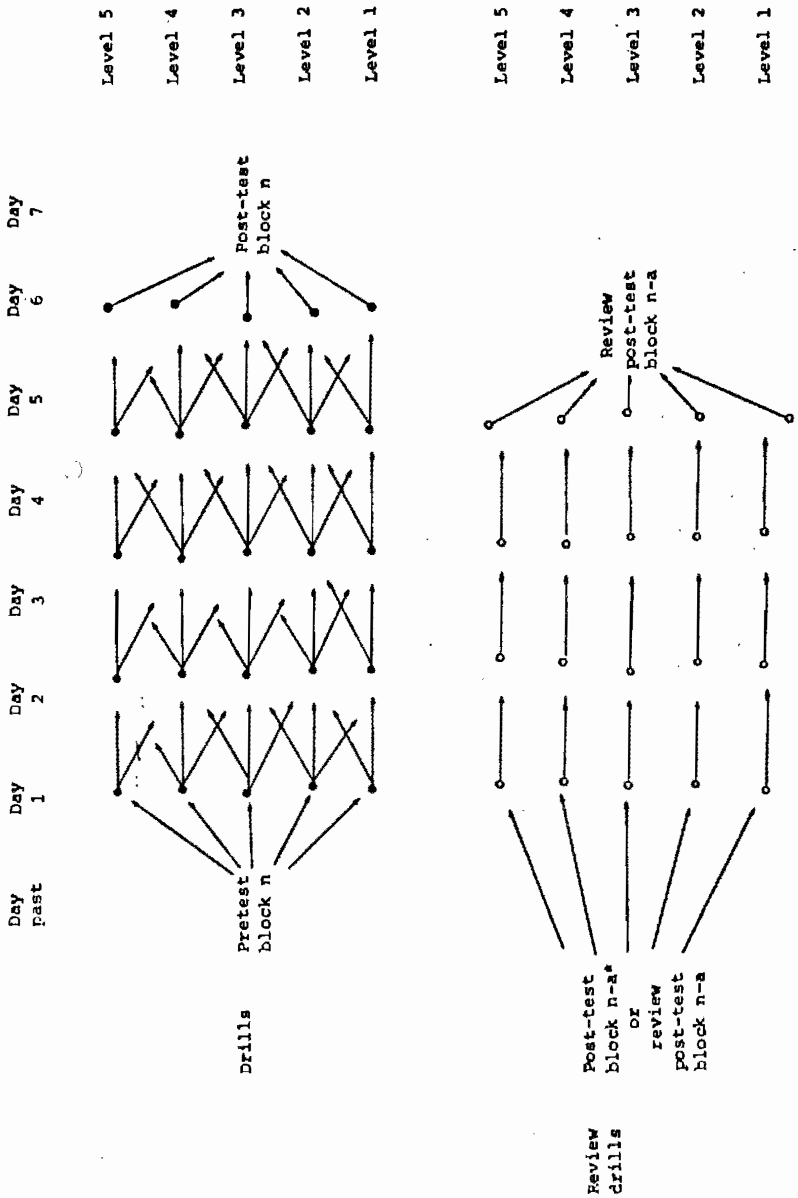

Fig. 2 Branching structure for a seven-day concept block.

Figure 12–2.
Branching structure for a seven-day concept block. *n-a block with lowest posttest performance.

Numbers, Book 1 (Suppes, 1965), with the addition of some topics such as oral story problems, which cannot, by their nature, be adapted to a textbook format. An outline of the programmed curriculum is shown in Table 12–3.

Table 12–3. Curriculum Outline for Grade 1—Brentwood Tutorial Mathematics Program

	Number of lessons			
Book	Core curriculum	Remedial branches	Drills	Description
1	8	1	0	Using the machine
2A	7	5	0	Introduction to sets
2B	3	3	0	Matching equal sets
3A	5	4	0	Union of two sets with one member
3B	3	1	2	Union of empty sets
4	11	4	1	Geometry—learning to identify squares, circles, triangles, and line segments
5	7	6	0	Balancing set equations
6A	8	7	0	Introducing the numerals 0, 1, and 2
6B	9	8	1	Introducing *N* notation with equivalent numerals, Introducing the numerals 3 and 4
7A	5	5	1	Sums with *N* notation (0–4)
7B	5	0	0	Sums with numerals, keyboard responses (0–4)
8	8	2	0	Review
9	8	7	1	Addition
10	9	2	1	Geometry
14A	4	0	1	Measuring line segments
14B	4	0	1	Concave figures, meaning of *half*
15	6	7	1	Balancing addition equations (0–9)
16	7	5	1	Number words, one–six
17A	6	5	2	Number words, zero–ten; sums to nine with three addends
17B	9	1	1	Subtraction
18A	11	6	0	Subtraction combinations through 6
18B	6	2	2	Relating addition and subtraction
19A	12	1	2	Geometry, matching similar figures
19B	11	3	1	Subsets
20A	4	3	2	Review
20B	6	3	2	Subtraction $c - a = b, 7 < c < 9$
21A	11	0	0	Counting and typing to 19
21B	6	2	0	Place value
22	17	6	1	Addition on the number line
23	14	1	1	Counting by 10's
24	19	2	2	Counting by 5's
25	14	2	1	Addition combinations (10–15)

Because the programmed lessons were tutorial, many of the lessons were explanatory, relying on oral explanations synchronized with changing visual displays. The lessons were short (the average length was less than ten problems) and explanations were simple and direct. Generally the problems within one lesson were all of the same type; the first few were accompanied by explanatory audio messages, leaving the remainder as practice problems.

The students received their programmed instruction in a room that contained seventeen student stations and a proctor station for use by the teachers on duty. The student stations were separated by four-foot partitions, which extended far enough from the walls to provide a degree of privacy to the students. When the children arrived in the student station room, they looked for their names on the CRT screen at their assigned stations, put on their headsets, and started their program by touching the light pen to a smiling face displayed on the CRT. After the allotted time for the class, approximately twenty minutes, the students were automatically signed off as they completed their current lesson; the message YOU HAVE BEEN SIGNED OFF appeared on the CRT and the children went into the adjoining classroom, joining a teacher who escorted them back to their homeroom.

Both explanatory and practice problems contained provisional audio messages that were heard only by students who responded incorrectly or who failed to respond within a reasonable time. For example, for one problem, a drawing of a car and a drawing of a truck surrounded by set braces and followed by an equal sign were presented on the CRT; this problem was accompanied by the audio message, "There are two members in this set." After this message, two more sets, one empty and one containing a train and a steam shovel, each preceded by a box, were displayed below the initial set; the choices were accompanied by the audio instructions, "Find another set with two members." At this point a small "p" (for pen) was displayed in the corner of the CRT as a signal to the student to respond. If the student touched his light pen to the box in front of the correct choice, a smiling face was displayed and he heard, "Yes, the sets have the same number of members," and proceeded to the next problem.

If the student did not respond within twenty seconds, he heard, "Which set below has two members?" If the student responded incorrectly, he heard the audio message, "Point to the box next to the set with two members," and saw a sad face. For most problems in the curriculum, students were allowed three chances to produce the correct answer. After three incorrect responses, the correct answer or an arrow pointing to the correct choice was displayed, accompanied by a brief audio message.

For most lessons the number of initial correct responses to the practice problems was accumulated and compared to a criterion. As soon as a student had made the required number of correct responses, he was allowed to skip remaining problems and begin the next lesson. As soon as a student failed criterion, which was, for instance, three incorrect answers if the criterion was seven out of nine problems, he was branched immediately to a remedial lesson containing the same kinds of problems but with a slower development of ideas using simpler vocabulary and sentence structure. If a student failed criterion on a remedial lesson, his program stopped and an automatic call for assistance from a proctor was typed at the proctor station.

This mode of branching permitted students to make their own progress through the curriculum. Thus all students were not required to work on the same concept block at the same time as in the drill-and-practice program. In fact, the faster students were considerably separated in terms of curriculum material from the slower students.

The classroom activity, completely coordinated with the programmed instruction, contained (1) use of physical objects to introduce concepts presupposed by the programmed lessons, (2) work originally planned as programmed lessons, (3) remedial work for individual children, and (4) enrichment material for individual or group use.

To summarize, the basic features of the Brentwood tutorial mathematics program were

1. Role—tutorial.
2. Age group—grade 1.
3. Curriculum—elementary mathematics.
4. Individualization:
   a. Temporal pace of problem presentation determined by the student.
   b. Immediate feedback.
   c. Opportunity for second response if first one incorrect with audio hints to aid problem solution.
   d. Opportunity to advance faster by eliminating some practice problems if student performance met criterion.
   e. Remedial problems at lower difficulty level if student below criterion performance.
   f. Special teacher attention if remedial performance below criterion.
5. External materials—teachers, proctors, all aids normally available in the classroom.
6. Number of students—forty-nine.
7. Presentation of curriculum—cathode-ray tube, film, and audio.
8. Response mode—light pen and standard keyboard.

The students' station room at the Brentwood Laboratory was shared with a reading curriculum program directed by Professor Atkinson (Atkinson and Hansen, 1966; Atkinson and Wilson, 1967). All the first graders not participating in the mathematics curriculum, approximately fifty students, spent about twenty minutes per day at the computer terminals on the tutorial reading curriculum. Both the reading and mathematics programs at Brentwood contained all the individualization inherent in the drill-and-practice program plus the additional features described.

Logic and Algebra Program

The logic and algebra program, similar to the Brentwood mathematics program, is tutorial. However, the teaching of symbolic logic and algebra, unlike the mathematics curriculum for Brentwood and the drill-and-practice program, introduces curriculum materials that are not encountered in a standard elementary school mathematics curriculum. Thirty students in the fourth grade started with sentential logic in December 1966; late in January 1967, the same students started algebraic derivations with logic and algebra presented on alternate days.

The logic curriculum introduced thirteen rules, shown in Table 12–4,

Table 12–4. Rules for Sentential Logic

FC—form a conjunction		
P	(1)	R
P	(2)	S
1.2 FC	(3)	R & S
LC—left conjunct		
P	(1)	R & S (conjunction)
1 LC	(2)	R
RC—right conjunct		
P	(1)	R & S (conjunction)
1 RC	(2)	S
CC—commute conjunction		
P	(1)	R & S
1 CC1	(2)	S & R
AA—affirm the antecedent		
P	(1)	R → S (conditional)
P	(2)	R (affirms antecedent)
1.2 AA	(3)	S
DC—deny the consequent		
P	(1)	R → S (conditional)
P	(2)	¬S (denies consequent)
1.2 DC	(3)	¬R
DN—double negation		
P	(1)	¬¬S (¬¬is dominant)
1 DN	(2)	S
		or
P	(1)	S (¬¬ is not dominant)
1 DN	(1)	¬¬S
CD—commute disjunction		
P	(1)	R v S
1 CD1	(2)	S v R
DD—deny a disjunct		
P	(1)	R v S (disjunction)
P	(2)	¬R (denies a disjunct)
1.2 DD	(3)	S
FD—form a disjunction		
P	(1)	R
1 FD	(2)	(R) v (S)[a]
HS—hypothetical syllogism		
P	(1)	R → Q
P	(2)	Q → S
1.2 HS	(3)	R → S
CP—conditional proof		
P	(1)	Q
WP	(2)	R (added premise)
	(3)	S
2.3 CP	(4)	R → S
IP—indirect proof		
P	(1)	Q
WP	(2)	R (added premise)
	(3)	S
	(4)	¬S
2.3.4 IP	(5)	¬R

[a] *Filled in by student.*

including conditional proof and indirect proof, presented in nineteen lessons averaging nineteen problems per lesson. Most of the problems were derivations of one or more steps to be carried out by the use of the thirteen rules. A multiple-choice response mode was used for vocabulary drill, dominance of connectives, strategies, and derivations involving English sentences. Beginning at the end of lesson 9, occasional use was made of the algebraic rules presented in the algebra program. The topics covered in the logic curriculum are presented in Table 12–5.

Table 12–5. Topics Covered in the First-year Logic Program

Lesson number	Rules intro-duced	Vocabular items introduced	Topics covered
1	AA	Conditional, antecedent derive	Deriving conclusions, extraneous premises
2			Additional one-step uses of rule AA
3			Two-step derivations with AA
4	FC	Conjunction	One- then more-than-one-step use of FC, FC and AA used in same derivations
5	LC, RC		Use of LC and RC and their use with AA and FC
6	DD	Disjunction	One- and two-step use of DD, use with the other rules
7	DC	Deny, consequent	Use of DC, its use with other rules
8	DN		DN to add, then to remove dominant double negation
9	HS	Hypothetical syllogism	Use of HS, use of the algebraic rule ND within sentential derivations
10	FD		Use of FD
11			Derivations with all rules presented, use of multiple-choice mode for a review of vocabulary and rule uses
12	CD, CC	Commute	Use of rules CD and CC, in multiple-choice mode English application
13			Review and English examples by multiple-choice mode
14			Dominance in complex cases
15	CP		Technique of conditional proof
16			Special rules for dominance without parentheses
17	IP		Technique of indirect proof
18, 19:	Two lessons introducing the multiple-choice mode, inserted into all student's curriculums as soon as the mode was programmed and the lessons coded		

The goal of the algebra curriculum was to teach the students the fundamental properties of the commutative ring of integers with unity and how to use these properties in formal mathematical proofs. The curriculum covered the properties of (1) commutativity of addition and multiplication, (2) associativity of addition and multiplication, (3) additive and multiplicative identities, (4) additive inverse, and (5) the distributive law. A total of fifteen rules, shown in Table 12–6, were presented during the course of twenty-three lessons with from eleven to thirty problems per lesson.

Table 12–6. Rules for the Algebra Program

Rule	Example	Line
ND—number definition	7ND	(1) 7 = 6 + 1
D—definition		(1) 6 = 5 + 1
	5D1	(2) 6 = (4 + 1) + 1
CA—commute addition		(1) 4 = 3 + 1
	CA1	(2) 4 = 1 + 3
AR—associate addition right		(1) 4 = (2 + 1) + 1
	AR2	(2) 4 = 2 + (1 + 1)
AL—associate addition left		(1) 4 = 2 + (1 + 1)
	AL1	(2) 4 = (2 + 1) + 1
ID—inverse definition		(1) 5 = 3 + (1 + 1)
	2ID1	(2) 5 = 3 + 2
CM—commute multiplication		(1) 6 = 2 × 3
	CM1	(2) 6 = 3 × 2
MR—associate multiplication right		(1) 12 = (4 × 1) × 3
	MR2	(2) 12 = 4 × (1 × 3)
ML—associate multiplication left		(1) 12 = 4 × (1 × 3)
	ML1	(2) 12 = (4 × 1) × 3
DL—distributive law		(1) A = 4 × (5 + 6)
	DL1	(2) A = (4 × 5) + (4 × 6)
Z—zero		(1) 7 = 7 + 0
	Z1	(2) 7 = 7
IZ—inverse zero		(1) 7 = 7
	7IZ2	(2) 7 = 7 + 0
N—negative		(1) 2 = 6 − 6 + 2
	6N1	(2) 2 = 0 + 2
IN—inverse negative		(1) 2 = 0 + 2
	6IN1	(2) 2 = 6 − 6 + 2
MI—multiplicative identity		(1) 6 = 6 × 1
	6M12	(2) 6 = 6

The students, as in the drill-and-practice program, were presented problems on a Model-33 teletype. The rules in the logic and algebra program were presented to the student one at a time, followed by a few problems making one-step and then two-step use of the rule. Thereafter, derivations made regular use of the new rule and all previously given rules. All students received the same curriculum. For multiple-choice problems the error message WRONG, TRY AGAIN followed incorrect responses; the student continued to respond until he gave a correct answer. For derivation problems, incorrect syntax or invalid commands from the student were followed by brief explanatory error messages.

Problem 13–5 is a typical example of a sentential derivation. The numbers in parentheses are line numbers; all input by the student is underlined.

Problem 13–5

```
YOU MUST DERIVE 'K v S' BEFORE USING CP.
  DERIVE J → K v S
  P           (1)   K v ¬J
                        J
  P           (2)       J
  DD1.2       (3)       K
  FD
                        NOT ENOUGH LINE NUMBERS GIVEN.
  FD3         (4)       (K) v (S)
  CP2.4       (5)   J → K v S
  CORRECT
```

The problem and the premises (P) on lines (1) and (2) were printed by the teletype. The student then typed DD1.2, which commanded that the *deny-a-disjunct* rule be applied to the disjunction in line (1) using the premise in line (2). The teletype then printed the result of the command including the line number. The error message between lines (3) and (4) informed the student why the rule FD, form a disjunct, was not executed. When the student typed the proper command, FD3, (K) V __ was printed. The student then formed the disjunction he wanted by typing (S). The application of the conditional-proof rule to lines (2) and (4) completed the derivation correctly.

An example of a completed problem from the algebra curriculum is

```
DERIVE          7 = (5 + 1) + 1
ND7       (1)   7 = 6 + 1
6D1       (2)   7 = (5 + 1) + 1
```

The ND7 typed by the student on line (1) calls for the number definition of 7. The program responds to this command by typing the line number and the result of applying the ND rule. The number preceding the D on line (2) gives the number to be replaced by its definition; the number following the D states which occurrence of that number is to be replaced. Thus 6D1 asks the program to replace the first occurrence of 6 on the previous line with its definition.

In addition to material on the teletype, the students received a handbook and an instruction booklet. The handbook defined all rules, defined kinds of formulas and dominance of connectives, and listed possible strategy questions. The instruction booklet presented each rule, emphasizing the mechanics of typing the correct information necessary to apply the rule. For each rule several examples were given showing the typed command and the result. Following the examples, from three to six problems were given in which the student was shown the result of an unknown command and told to fill in the missing command.

Although this program contained no branching as a function of performance, the student could proceed through the curriculum at a pace congruent with his ability. For most of the derivations a unique solution did not exist, and each student could develop his own solution strategy. If a student encountered difficulties, a staff member was present to give on-line assistance.

To summarize, the basic features of the logic and algebra program are

1. Role—tutorial.
2. Age group—grade 4.
3. Curriculum—logic and algebra.
4. Individualization:
   a. Temporal pace of curriculum presentation determined by the student.
   b. Immediate feedback.
   c. Opportunity to correct errors immediately.
   d. Student could develop his own strategy for solving problems.
5. External materials—handbook, instructional booklet, attending staff member.
6. Number of students—30 in 1966–1967 (more than 200 in 1967–1968).
7. Presentation of curriculum—teletype.
8. Response mode—keyboard (with some extra logical symbols).

Russian Program

The Russian program was instituted at Stanford in September 1967. This project is directed by Dr. Joseph Van Campen, who has designed a program to teach the standard aspects of a first-year course at the college level: comprehension of written Russian, comprehension of spoken Russian; and mastery of grammar and syntax. Of the three main components of a college-level language course, regular classroom sessions on a daily basis, time in the language laboratory, and regular homework assignments, only the functions of the tutorial classroom sessions have been assumed by the computer program. In addition to their time at the computer console, the students have to spend time in the language laboratory and do homework assignments. The language-laboratory tapes with drill sheets and the homework assignments are prepared by the staff at the Institute.

The twenty-nine students beginning the introductory Russian course during the fall quarter of 1967 were required to spend about fifty minutes per day, five days per week, at the computer console. A total of 135 lessons were prepared, which were presented to the students in a combined audio and teletype format. The students responded on a Model-33 teletype with a special keyboard containing the Cyrillic alphabet.

During the first quarter all errors by the student were corrected immediately. Because typing errors were corrected by the computer just as if they were real mistakes on the student's part, a distorted picture of performance occurred for problems with extensive amounts of typing. To correct this distortion, a change was made in the second quarter that allowed the student to (1) delete as many characters as he wished, replacing them with "correct" characters, or (2) delete the entire answer and retype from the beginning. When he felt the response was correct, the student pressed a special key that instructed the computer to check the given item.

Although the basic curriculum was the same for all students, there were several remedial branches. At given points students were tested on several items of a given type and were given remedial instruction on the points covered if their performance on the test block failed to meet a satisfactory standard. Later in the year routines were provided that produced more specific remedial work based on the type of error, a sophisticated approach to remedial work not present in any of the other programs.

The educational level of the students in the Russian program permitted another unique role for the computer. During the period prior to the final examination, lesson summaries for each new lesson and a final summary covering the material for the entire quarter were given to the students. The computer then assessed the student's performance and told him the rules on which he should concentrate his efforts. At following sessions the student was again tested on the points he had missed and informed where more study was needed. In addition, the student could redo any lesson or portion of a lesson at the computer console.

The language-laboratory tapes provided material for pronunciation practice and also for testing a student's ability to comprehend spoken Russian. A test at the end of the tape either presented a number of Russian sentences for transcription by the student or required the student to respond in writing to oral questions on a paragraph that he had just heard.

To evaluate the pronunciation of the students, a process not possible at this point on the computer, two recordings were made each quarter. The students were counseled immediately after each recording as to what pronunciation errors were made and how to correct them.

To summarize, the basic features of the Russian program are

1. Role—tutorial.
2. Age group—college.
3. Curriculum—Russian.
4. Individualization:
   a. Immediate feedback.
   b. Remedial blocks when performance below standard.
   c. Directed review for examinations.
   d. Could repeat lessons on own initiative.
5. External materials—language-laboratory tapes and tests, homework assignments, individual counseling on pronunciation.
6. Number of students—twenty-nine.
7. Presentation of curriculum—audio and teletype.
8. Response mode—keyboard (with Cyrillic alphabet).

RESULTS AND DISCUSSION

The amount of data collected and the extensive number of analyses performed prohibit a complete presentation of results in this chapter. Consequently, we restrict ourselves to specific examples from each of the programs.

Drill-and-Practice Program

The major emphasis in our analysis of data from the drill-and-practice program was to delineate the factors contributing to problem difficulty. Multiple-regression models with average proportion correct as the dependent variable were applied to the data. Analysis of the pretests and posttests from two blocks of subtraction problems from the fourth grade, 402 and 403, will serve as examples of this technique.

Block 402 contained both horizontal and vertical subtraction problems; all the problems in block 403 were in a vertical format. Two of the independent variables, DIFF and BOR, accounted for characteristics of the numbers shown above the line in vertical problems and to the left of the equal sign in horizontal problems. The two other independent variables, VF and HVC, depended partly on the characteristics of the correct response. Noncanonical horizontal problems, for example, $36 - \_\_ = 30$, were treated as if the missing number were present.

In the following definitions, lowercase letters represent digits. The four independent variables were

1. Differences (DIFF: possible values, 0–4). The value of DIFF for

each problem was equal to the number of columns with the exception of basic borrow problems. The basic borrow problems are those of the form $ab - c = \_\_$, where the minuend (ab) is a number from 10 to 18. Although these problems have two columns, they involve only one difference and therefore received a value of 1 for this variable.

$$\text{DIFF}\begin{pmatrix} a \\ -b \\ \hline c \end{pmatrix} = 1$$

$$\text{DIFF}\begin{pmatrix} ab \\ -c \\ \hline d \end{pmatrix} = 1$$

$$\text{DIFF}\begin{pmatrix} abc \\ -de \\ \hline fgh \end{pmatrix} = 3$$

2. "Borrows" (BOR: possible values, 0–2). The value of BOR for each problem was the number of times a 1 was "borrowed" from an adjacent column, or, in more recent language, the number of times a regrouping was required.

$$\text{BOR}\begin{pmatrix} ab \\ -cd \\ \hline ef \end{pmatrix} = \begin{cases} 0 & \text{if } b \geqq d \\ 1 & \text{if } b < d \end{cases}$$

$$\text{BOR}\begin{pmatrix} ab \\ -c \\ \hline d \end{pmatrix} = 1$$

$$\text{BOR}\begin{pmatrix} abc \\ -def \\ \hline ghi \end{pmatrix} = \begin{cases} 0 & \text{if } c \geqq f,\ b \geqq e \\ 1 & \text{if } c < f,\ (b-1) \geqq e \\ 1 & \text{if } c \geqq f,\ b < e \\ 2 & \text{if } c < f,\ (b-1) < e \end{cases}$$

3. Vertical format variable (VF: possible values, 0, 1, 3). All horizontal problems and vertical problems with one-digit responses received a value of 0. Multicolumn problems with multidigit responses and one-column addition problems with a sum of 11 received a value of 1. One-column addition problems with a multidigit sum other than 11 received a value of 3.

$$\text{VF}\begin{pmatrix} ab \\ -cd \\ \hline e \end{pmatrix} = 0$$

$$\text{VF}\begin{pmatrix} abc \\ -def \\ \hline ghi \end{pmatrix} = 1$$

VF probably reflects the likelihood of the mistake of reversing the digits of the correct response. Responses to vertical problems were typed from right to left, whereas responses to horizontal problems were typed from left to right. Thus a student could know the correct answer but err by typing the digits in the reverse order.

4. Horizontal format composite variable (HFC: possible values, 0–3). Horizontal problems were assigned a 1 for each of three characteristics. The sum of the 1's for each problem was the value of HFC. Ones were assigned to problems in noncanonical form, to problems in which the minuend and the subtrahend or the addends were each two or three digits, and to problems with multidigit responses.

$$\begin{aligned} \mathrm{HFC}(a - b = \underline{c}) &= 0 \\ \mathrm{HFC}(ab - \underline{c} = de) &= 1 \\ \mathrm{HFC}(ab - cd = \underline{ef}) &= 2 \\ \mathrm{HFC}(\underline{ab} - cd = ef) &= 3 \end{aligned}$$

Although the examples given for assigning values of variables to the problems are all subtraction problems, VF and HFC also apply to addition problems. Two other independent variables, not presented here, were defined and, in combination with the four presented, were used in the analysis of all data from addition and subtraction problems in the third, fourth, and fifth grades.

The correlation coefficient (R), the coefficient of determination (R^2), and the regression coefficients for the four variables are shown in Table 12–7 for the pretests and posttests on blocks 402 and 403. When the same problem occurred more than once in a block, the data for the identical problems were averaged. There were forty-eight problems in block 402 and sixty problems in block 403.

Table 12–7. Regression Coefficients for Grade 4 Subtraction—Drill-and-Practice Program

		Number of subjects				Variables					
Block	Test	Form A	Form B	Form C	Constant	DIFF	BOR	HFC	VF	R	R^2
402	Pre	64	69	81	−3.49	0.49	0.98	1.28	1.33	0.94	0.87
	Post	75	49	68	−3.63	0.80	0.74	0.86	0.56	0.85	0.72
403	Pre	56	65	77	−2.75	0.06[a]	1.26		0.22[a]	0.89	0.79
	Post	69	43	54	−2.75	−0.01[a]	0.86		0.52[a]	0.70	0.49

[a] Not significant.

Several of the results presented in Table 12–7 are similar to those obtained for all the addition and subtraction blocks we analyzed. First, the correlation coefficient decreases from pretest to posttest. Second, the regression coefficients for the independent variables, with few exceptions, decrease from pretest to posttest. Finally, the vertical format variable is significant for the block with both horizontal and vertical format (402); it is not significant in the block with only vertical problems (403). This last finding supports the interpretation that VF reflects the likelihood of typing the digits of the response in reverse order, because this error would be more probable in mixed-format blocks than in blocks with all vertical problems.

To provide a description of performance in terms of these variables, the problems in block 402 were classified into similar types. Problems that received the same pattern of values on the four variables were considered identical. Table 12–8 presents the number of problems, the mean observed proportion correct, the predicted probability correct, and the average deviation between predicted and observed for each problem type.

Table 12–8. Observed and Predicted Probabilities for Problem Types, Block 402 —Drill-and-Practice Program

Problem type	Number of problems	Average observed probability	Predicted probability	Average deviation
Horizontal format				
$a - b = \underline{c}$[a] No borrow	4	0.96	0.95	0.01
$ab - c = \underline{d}$ One borrow	2	0.92	0.86	0.04
$\underline{ab} - c = d$ One borrow	3	0.42	0.37	0.05
$ab - c = \underline{de}$ No borrow	3	0.80	0.77	0.03
$ab - \underline{c} = de$ No borrow	6	0.78	0.77	0.01
$ab - c = \underline{de}$ One borrow	3	0.44	0.56	−0.12
$\underline{ab} - c = de$ One borrow	3	0.24	0.26	−0.02
Vertical format				
$a - b = \underline{c}$ No borrow	2	0.92	0.95	−0.03
$ab - c = \underline{d}$ One borrow	1	0.87	0.88	−0.01
$ab - c = \underline{de}$ $ab - cd = \underline{ef}$ No borrow	9	0.75	0.77	−0.02
$ab - c = \underline{de}$ One borrow	9	0.53	0.55	−0.02
$abc - de = \underline{fgh}$ One borrow	3	0.52	0.43	0.09

[a] *Underlined digits are the students' response.*

In this block the predicted probabilities were usually lower than the observed ones for horizontal problems but were higher than the observed for vertical problems. In general, the average deviation was low. The exception for horizontal problems was the format $ab - c = \underline{de}$ with one "borrow." For vertical problems the largest deviation occurred for the problem $abc - de = \underline{fgh}$ with one borrow.

Extensive regression analyses of the kind presented here have already been published (Suppes et al., 1968), using the data from our 1965–1966 program. Although it is not possible to enter into details, the analyses of the 1965–1966 and 1966–1967 data were used directly in making revisions in the drill-and-practice program for 1967–1968. Still more extensive revisions are being made during 1968–1969. In fact, for 1968–1969 we are abandoning the block structure described earlier and using concept strands that run across a number of grades. Each student's position, as represented by a vector of grade placements in the strands, depends only on his own work and progress. A preliminary description of the strand approach is to be found in Suppes (1967).

Brentwood Tutorial Mathematics Program

Three approaches were followed in the analysis of the Brentwood data. First, regression models were used to relate structural properties of problems to proportion correct and success latency when performance on a given problem was an average of all students completing that problem. Second, regression models were utilized to identify significant factors in individual performance. Finally, data were compiled to describe performance, in terms of proportion correct, as a function of the problem types defined by the factors found significant in the structural analyses.

Individual models. The students completed 270 lessons. Each lesson contained from 10 to 15 problems. A block, the unit on which performance was measured, contained from 25 to 100 sequential problems on the same concept. When one or two lessons on one concept were interspersed in a series of lessons on another concept, the interspersed lessons were not included in the analysis. Thus no problems in block n were completed prior to the completion of the problems in block n-1. If a sequence of lessons on the same concept contained more than 100 problems, a new block was formed. In this manner thirty-nine blocks were formed on the concepts of addition, subtraction, sets, geometry, counting, numerals, sequences, and miscellaneous. About 70 per cent of the original lessons were used in the final analysis.

Two basic types of models were examined: a temporal model in which the prediction of an individual's performance in a given block was based on his performance in the immediately preceding blocks and a conceptual model in which prediction was based on performance on previous blocks of the same concept. A number of the models we examined are presented in Table 12–9.

Table 12–9. Models for Prediction of Individual Performance—Brentwood Tutorial Mathematics Program

Type	Model[a]	Parameter estimation
I Temporal	$p_{s,i} = \alpha_i + \beta_i p_{s,i-1} + \theta_i p_{s,i-2}$	Group
II Temporal	$p_{s,i} = \alpha_s + \beta_s p_{s,i-1} + \theta_s p_{s,i-2}$	Individual
III Temporal	$p_{s,i,m} = \alpha_{s,m} + \beta_{s,m} p_{s,i-1}$	Modified individual; estimated every four blocks
IV Temporal-conceptual	$p_{s,i,c} = \alpha_i + \beta_i p_{s,i-1} + \theta_i p_{s,i-k}$	Group
V Temporal-IQ	$p_{s,i} = \alpha_i + \beta_i p_{s,i-1} + \theta_i IQ_s/100$	Group
VI Conceptual	$p_{s,i,c} = \alpha_i + \beta_i p_{s,i-k} + \theta_i p_{s,i-j}$	Group
V Conceptual	$p_{s,i,c} = \alpha_{s,c} + \beta_{s,c} p_{s,i-k}$	Modified individual; estimated for each concept

[a] *Where p is proportion correct; s is the student; i is the block, α, β, and θ are parameters to be estimated; m is a set of four blocks; c is a concept; i - k is the block immediately preceding i with same c; and i - j is the block immediately preceding i - k with same c.*

Because the difference in an individual's performance at two points in time is a function of the curriculum and of individual differences, three methods of parameter estimation were employed. If one assumes that the major factor in an individual's performance is the curriculum, then group-parameter estimation is most appropriate. For each of the group-parameter estimation models (Table 12–9), one set of parameters was estimated for each block, using the data from all students in the blocks appropriate to the model. Thus, to predict an individual's performance, a different set of parameters was used for each block; for a given block the same set of parameters was used for all individuals.

The individual-parameter estimation technique assumes that differences within the individual are dominant. Under this assumption the performance data on all blocks for a given individual were utilized to estimate parameters. Thus, to predict a given individual's performance, a set of parameters unique to the individual was used for all blocks. For a given block a different set of parameters was used for each student. A more realistic combination of curriculum and individual effects resulted in a modified individual-estimation technique where all data from a given student's performance on a subset of blocks yielded a set of parameters. Thus the set of parameters used to predict an individual's performance was unique to the individual and to a subset of the curriculum. Both temporal (model III) and conceptual (model VII) subsets were employed in the modified individual-estimation procedure.

Because the number of students completing a block decreased rapidly after block 35, only the data from the first thirty-five blocks were used for the estimation of group parameters. All blocks completed by a given student were used in the estimation of parameters for that student. The number of blocks completed varied from twenty-three to thirty-nine.

The regression program used for parameter estimation yielded a chi-square value based on the observed performance data and the predicted value determined by the multiple regression equation for each student for each model. The number of parameters estimated and the total number of predictions for each model are shown in Table 12–10. The average chi-squares in Table 12–10 were calculated by averaging the individual chi-

Table 12–10. Comparison of Models for Predicting Individual Performance—Brentwood Tutorial Mathematics Program

Model	Average chi-square	Number of blocks	Number of individual estimates	Number of parameters estimated
I Temporal group	2.66	33	1199	99
II Temporal individual	4.33	35	1211	120
III Temporal modified individual	2.64	35	1211	598
IV Temporal-conceptual group	2.24	25	858	75
V Temporal-IQ group	2.66	33	1199	99
VI Conceptual group	2.26	20	699	60
VII Conceptual modified individual	1.56	27	938	480

squares for a given block and then averaging these averages to yield an average individual chi-square for each model. It must be noted that these chi-squares are for comparative use only; they are not corrected for number of parameters estimated and therefore should not be utilized for statistical inference.

Of these models the conceptual model with individual parameters for each concept (model VII) had the lowest average chi-square; the temporal model with individual parameters (model II) produced the largest chi-square. This analysis indicates that a student's performance on a given concept depends more on his past performance on the same concept than on his more recent performance on a different concept.

Although the conceptual model with modified individual parameters (model VII) appears to be the best model, the large number of parameters affects the goodness of fit. Two models using about the same number of parameters, the temporal-conceptual (model IV) and the conceptual model with group parameters (model VI), yielded very similar average chi-squares. This result indicates that the most recent performance was as useful in predicting performance as the most recent performance on a concept if the next most recent performance on that concept was used. However, the poorer fit for the temporal model with group parameters (model I) indicates that at least one conceptual factor is important.

The average chi-square for each model within each block provides evidence supporting the use of modified individual parameters. In this comparison the temporal model (III) with modified individual parameters yielded the lowest or second lowest chi-square in twenty-five of the thirty-five blocks, whereas the chi-square for the conceptual model (VII) was one of the two lowest for eighteen of the twenty-seven blocks for which predictions existed. The superiority of model III in this comparison appears contrary to the finding presented in Table 12–10. However, three blocks, two addition and one subtraction, were each more difficult than the other three blocks used in their four-block subset for estimation. Therefore, the observed performance was much lower than the predicted performance for model III, resulting in three of the highest chi-squares for any model in any block. These three large chi-squares contributed 47 per cent of the total chi-square for the thirty-five blocks, contributing disproportionately to the average presented for model III in Table 12–10. However, the superiority of these two models is not as genuine as it seems, because many more parameters were estimated for them than for the other five models.

Of additional note is the comparison between model I and model V. Given the most recent performance in time, the information added by either the next most recent performance or the IQ for the student made no difference in prediction as measured by average chi-square. The average chi-squares for the two models were also similar for all comparisons within blocks.

To summarize these models, conceptual factors predict performance better than temporal factors and, in terms of parameter estimation, modified individual techniques are better than group estimates. These, in turn, are better than individual-parameter estimation.

Group performance. To describe the absolute performance level of the students in the first grade mathematics curriculum, problems were grouped as a function of concept. Within each concept the mean proportion correct was examined as a function of the structural properties of the problems. Some examples of possible curriculum interest are given here. The concepts were (1) sets A, identity of sets and union of sets in canonical form; (2) sets B, identity of sets, union of sets in canonical and noncanonical form, and subtraction of sets; (3) geometry; (4) counting; (5) addition; and (6) subtraction. The mean proportion correct for each concept is shown in Table 12–11. Overall, the students performed best in geometry,

Table 12–11. Average Proportion Correct for Concepts—Brentwood Tutorial Mathematics Program

Concept	Proportion correct	Number of problems
Sets A	0.89	94
Sets B	0.74	62
Geometry	0.94	46
Counting	0.87	275
Addition	0.78	185
Subtraction	0.77	123

the identification of geometric shapes; their lowest performance was on the set problems appearing later in the curriculum (sets B).

Table 12–12 presents the proportion correct for addition problems for two of the factors examined, story versus nonstory and type-of-problem format. Children had the most difficulty with problems in noncanonical format, for example, 3 + ___ = 8.

A more extensive report of the Brentwood tutorial program will be subsequently published. Major aspects of the project have not been covered here, especially the operational or evaluation aspects. We have attempted to give a sense of the kind of detailed questions that can be asked (and answered) about individual student differences and about individual concepts in the curriculum.

Table 12–12. Average Proportion Correct for Addition Problems as a Function of Structural Characteristics—Brentwood Tutorial Mathematics Program

Structural characteristic	Proportion correct	Number of problems
Story versus nonstory		
Story	0.87	29
Nonstory	0.76	156
Type-of-problem format		
Vertical, 2 addends	0.82	9
Horizontal, 2 addends	0.79	64
Horizontal, 3 addends	0.64	12
Vertical, 3 addends	0.54	8
Horizontal, noncanonical	0.29	5

Logic and Algebra Program

We have restricted presentation of the analysis of data for this program to one important example. The rank ordering of seventeen logic and algebra rules of inference, in terms of mean latency, is shown in Table 12–13.

Table 12–13. Rank Ordering of Algebraic and Logical Rules of Inference on the Basis of Mean Response Latency

Rank	Rule	Latency, seconds	Number of occurrences of rule in data
1	ND (number definition)	2.93	2589
2	D (definition)	2.94	4732
3	CA (commute addition)	3.42	2885
4	LC (left conjunct)	4.06	25
5	AR (associate addition right)	4.09	1419
6	AL (associate addition left)	5.36	103
7	CD (commute disjunction)	5.75	41
8	ID (inverse definition)	6.11	361
9	AA (affirm the antecedent)	6.24	482
10	FD (form a disjunction)	6.34	325
11	DN (double negation)	7.12	244
12	CC (commute conjunction)	8.42	41
13	DC (deny the consequent)	9.16	302
14	DD (deny a disjunct)	11.73	219
15	HS (hypothetical syllogism)	11.79	137
16	FC (form a conjunction)	12.63	33
17	RC (right conjunct)	13.07	19

These are the seventeen rules used in the first half of the course. It is probably a fair inference to hold that the mean latency for each of the rules, which is based on summation over the students and occurrences of the rules in the student's proofs, is a good measure of relative difficulty. Perhaps the most important observation to be made about the data of Table 12–13 is that the six algebraic rules of inference are among the first eight rules in rank ordering. The definitional rule ND had a mean latency of 2.93 seconds and the longest latency for any of the algebraic rules was for the inverse rule ID, which had a mean latency of 6.11 seconds. Before a hasty inference is made to the conclusion that algebraic rules of inference are easier than those of sentential logic, it is important to keep in mind that the six algebraic rules each require reference only to a single preceding line, and in the present context this reference to the preceding single line was restricted to the *immediately* preceding line, which simplifies very much the search procedure the student must go through in deciding what rule to apply and where. The greater ease of application of the algebraic rules does suggest, however, that it would be wise to emphasize these rules at the beginning of the course. The data do show, too, that mastery of these algebraic rules is probably easier for students than mastery of the beginning rules of sentential logic.

The number of occurrences of each rule in the data, as summed across students and problems, is also shown in Table 12–13. We believe that the large discrepancy between the mean latency for LC and RC, which are conceptually so closely related, is not to be taken seriously in the present data because there were only twenty-five occurrences of the use of LC and only nineteen of RC. The logical rule CD also had only forty-one occurrences in the data. If we exclude LC and CD, no other rule of logic had a mean latency as short as any of the six rules of algebra. On the other hand, as might be expected, the rule of logic that follows immediately after the algebraic rules, with the exception of LC and CD, is AA, which is classical *modus ponendo ponens* and which had by far the most frequent occurrence in the data. Four of the six algebraic rules had very frequent occurrence, being used a good many times more than even the most frequent logic rule. This circumstance undoubtedly also helped produce the shorter latencies for the algebraic rules.

Russian Program

Although data analysis for the Russian program is not complete, several points of information are available. Thirty students started the autumn quarter in the computer-based Russian section. A total of eight students left the program—one during the first quarter, three between the first and second quarter, one during the second quarter, and three between the second and third quarter. Two new students entered the computer-based section at the beginning of the second quarter. Of the thirty-eight students enrolled for the autumn quarter in the regular Russian section, ten left the course during the first quarter, thirteen between the first and second quarter, and three between the second and third quarter. Four new students entered the regular section at the beginning of the third quarter, one of them a transfer from the computer-based class. Of the thirty students originally enrolled in the computer-based program, twenty-two (73 per cent) finished all three quarters, whereas of the thirty-eight students in the regular class, only twelve (32 per cent) finished the year's curriculum. This finding suggests that the interest of the students was maintained during the computerized teaching.

To evaluate the computerized program, approximately 66 per cent of the final examinations for the autumn and winter quarters were identical for the computer-based and for the regular Russian sections; the complete final examination for the spring quarter was identical for the two groups. The error distribution and the mean number of errors per student for the two groups on the final examination for the autumn, winter, and spring quarters are shown in Tables 12–14, 12–15, and 12–16, respectively. Although no significance tests have been applied to the differences in performance between the two groups, the average number of errors was lower for the

Table 12–14. Error Distribution for the Common Portion of the Autumn Quarter Final Examination—Russian Program

Number of errors	Number of students	
	Computer-based	Regular
3.5	1	
5	2	1
6	3	
7	1	
8	2	
9	3	
11	3	
13		1
15	1	
16	1	1
17	2	
19		1
21	2	1
22	1	1
23		2
25	1	1
27	3	
29		1
30		1
31		2
33	1	
34		1
37	1	
38	1	
41		1
43		1
45		1
53		1
61		1
64		1
65		1
72		1
76		1
79		1
93		1
97		1
120		1
141		1
Total number of students	29[a]	28[b]
Average number of errors	15.8	49.0

[a] Of the thirty students enrolled, one left during the quarter.
[b] Of the thirty-eight students enrolled, ten left during the quarter.

Table 12–15. Error Distribution for the Common Portion of the Winter Quarter Final Examination—Russian Program

Number of errors	Number of students: Computer-based	Number of students: Regular
2	1	1
6	1	
6.5	1	
8	1	
9.5		1
10	1	
11		1
12	2	
13		1
14.5		1
16	1	
16.5	1	
18	1	
18.5	1	
19	1	
19.5		1
21	2	
22.5	1	1
23	1	1
23.5	1	
24	1	
24.5		1
25	1	
26.5		1
27	1	
29.5	1	
30		1
30.5		1
32.5		1
33	1	
37.5	2	
38	1	
39.5	1	
41	1	2
47.5		1
Total number of students	27[a]	16[b]
Average number of errors	21.8	24.2

[a] *Three of the original students did not enroll, two new students were added, and one student did not finish the quarter.*
[b] *Thirteen of the original students did not enroll.*

computer-based students in all three quarters. Because the selection process resulting from the poorer students leaving the regular course biases the results on the examinations against the computer-based group, the superiority of the computer-based group on the spring examination is more impressive than the difference indicated by the average number of errors.

Table 12–16. Error Distribution for the Spring Quarter Final Examination—Russian Program

	Number of students	
Number of errors	Computer-based	Regular
21.5	1	
24.5	1	
26	1	
27	1	
31.5	1	
32	1	
34		1
35	1	
37	1	1
39		1
40	1	
41		1
42	1	
45	1	
46		1
47.5	1	
50.5		1
51.5	1	
60	1	1
61	1	
63.5		1
67		1
69	1	
69.5	1	
73	1	
74.5	2	1
76.5	1	
80.5		1
81	1	
82	1	
89		1
91		1
92		1
93	1	
106		1
166		1
Total number of students	24[a]	16[b]
Average number of errors	53.0	71.1

[a] Three students did not enroll.
[b] Three students did not enroll, three students enrolled for the first time, and one student transferred from the computer-based section to the regular section.

CONCLUSIONS

We have attempted to describe four programs in computer-assisted instruction and to give a sense of some of the results that have been obtained. The programs and the data flowing from them are complex and continuing.

We shall not yet attempt to give an overall summary of what we judge to be the significance of these programs. We do hope that we have included enough detail to give the reader a sense of our current activities in computer-assisted instruction and the kind of data results and analyses with which we have been concerned.

By presenting some programs being run in elementary schools and one program being run at the university level, we have also hoped to give a sense of the range of possibilities open for the use of computers as instructional devices. It is fair to say that during the early years of the Stanford operation we have concentrated on developing programs that can be run with fairly large numbers of students. This is particularly true of the drill-and-practice program in elementary mathematics. It is interesting to note, however, that even within the context of this program we are still in the process of making radical changes in it. The kind of data analysis reported here has been the source of much discussion among our staff regarding better ways to approach the subject. We feel that it will be still a good many years before even the possibilities in drill and practice are thoroughly developed, particularly as we move toward mathematical and quantitative formulations of the basic curriculum structure. From our current work it is evident that without the kind of empirical information reported here it will not be possible to move to more mathematically and scientifically sophisticated formulations of the curriculum.

So we would like to conclude on the note that we consider none of the programs reported here as being in a final form. Each of them is in process of development and change. What we have attempted to give are some examples that constitute an interim report. It is far too soon to attempt anything like an overall set of final conclusions.

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