TEST OF SOME LEARNING MODELS FOR DOUBLE CONTINGENT REINFORCEMENT

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Test of Some Learning Models for
Double Contingent Reinforcement[1]

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By 'double contingent reinforcement' we mean a reinforcement schedule in which the probability of a reinforcement on a given trial depends on the responses made on that trial and the preceding trial. From a theoretical standpoint such a schedule is of interest for several reasons. It is one of the simplest for which there do not exist simple recursions in the mean probability of response for the linear models of Bush and Mosteller (1955) or Estes and Suppes (1959a). Secondly, the standard stimulus sampling models must be modified to be applicable. In particular, the definition of state for the Markov chain cannot be the state of conditioning on trial n, nor even the pair of states of conditioning on trial $n-1$ and trial n. (For extensive discussion of the matter, see Estes and Suppes (1959b).)

In what follows we restrict ourselves to two-response situations. The double contingent reinforcement probabilities π_{ij} are the conditional probabilities $P(E_{1,n}|A_{j,n}A_{i,n-1})$ where $A_{j,n}$ is response j on trial n, for $j = 1,2$, $E_{1,n}$ is the reinforcement of response 1 on trial n, etc. As should be apparent, $P(E_{2,n}|A_{j,n}A_{i,n-1}) = 1 - \pi_{ij}$. For the experiment reported in this paper

$$\pi_{11} = .4 \qquad \pi_{21} = .2$$
$$\pi_{12} = .9 \qquad \pi_{22} = .7 \tag{1}$$

We now consider several different theoretical models, all of which fall within the general framework of stimulus sampling theory.

One-element θ model.--The basic assumptions embodied in this model are the following. The set of stimulus elements determining S's response on each trial has exactly one element which, in the present case, may be interpreted as the signal light occurring at the onset of each trial. This single stimulus element is sampled by S on each trial with probability 1 and becomes conditioned, if it is not already, to the reinforced response with probability θ. This conditioning probability is independent of the trial number and the outcome of preceding trials. (For detailed discussion of this model, together with many experimental applications, see (Suppes and Atkinson (1960).)

The appropriate definition of state in the multi-element model is the state of conditioning on trial n and the response on trial $n-1$. In the one-element model this reduces to the pair of responses on successive trials. By standard methods the following transition matrix is easily derived.

	$A_{1,n}A_{1,n+1}$	$A_{1,n}A_{2,n+1}$	$A_{2,n}A_{1,n+1}$	$A_{2,n}A_{2,n+1}$
$A_{1,n-1}A_{1,n}$	$1-(1-\pi_{11})\theta$	$(1-\pi_{11})\theta$	0	0
$A_{1,n-1}A_{2,n}$	0	0	$\pi_{12}\theta$	$1-\pi_{12}\theta$
$A_{2,n-1}A_{1,n}$	$1-(1-\pi_{21})\theta$	$(1-\pi_{21})\theta$	0	0
$A_{2,n-1}A_{2,n}$	0	0	$\pi_{22}\theta$	$1-\pi_{22}\theta$

From this matrix we derive at once the mean asymptotic probabilities

$$
\begin{aligned}
P_\infty(A_1A_1) &= \pi_{22}[1-\theta(1-\pi_{21})]/D \\
P_\infty(A_1A_2) &= P_\infty(A_2A_1) = \theta\pi_{22}(1-\pi_{11})/D \qquad (2)\\
P_\infty(A_2A_2) &= [1-\pi_{11}+\theta\pi_{12}(\pi_{11}-1)]/D \quad ,
\end{aligned}
$$

where

$$D = (1-\pi_{11}+\pi_{22}) + \theta[\pi_{22}(1-2\pi_{11}+\pi_{21}) + \pi_{12}(\pi_{11}-1)] \quad .$$

Also

$$P_\infty(A_1) = \pi_{22}[1+\theta(\pi_{21}-\pi_{11})]/D \quad . \qquad (3)$$

<u>Generalized conditioning models</u>.--This class of models still assumes that only one stimulus element is available for sampling, but generalizes the conditioning assumptions in several directions. To investigate various possibilities, we consider two subclasses. In one, G.C.M.I., the most general model supposes that the probability of conditioning, independent of the state of conditioning at the start of the trial, depends on the two preceding responses and reinforcements. Thus the conditioning parameters $c_{i'j'ij}$ are the conditional probabilities $P(A_{1,n+1}|E_{j,n}A_{i,n}E_{j',n-1}A_{i',n-1})$. Five special cases of this general model are considered in order to study the efficacy of various kinds of past information in predicting responses. The special cases are defined by restricting the dependence of $A_{1,n+1}$ to: (1) the response and reinforcement that occurred on trial n; (2) the two preceding reinforcements; (3) the two preceding responses; (4) the two preceding reinforcements and the immediately preceding response; (5) the two preceding responses and the immediately preceding reinforcement.

In the other subclass, G.C.M. II, the conditioning parameters are defined, not in terms of the sides 1 and 2, but in terms of successful and unsuccessful responses, rewarding and punishing reinforcements (successful prediction of a reinforcement makes it rewarding), repetition or alternation responses, or anticipation of a repeating or alternating reinforcement. Five special cases are defined by: (1) the reinforcement on trial n is punishing or rewarding; (2) the reinforcements on trials $n-1$ and n are punishing or rewarding; (3) the reinforcement on trial n is punishing or rewarding, and the response on trial n is anticipation of a repeating or alternating reinforcing event-- thus four conditioning parameters are estimated; (4) the reinforcement on trial n is punishing or rewarding, and the response on trial n is a repetition or alternation of the response on trial $n-1$; (5) the reinforcements on trials $n-1$ and n are punishing or rewarding, and the response on trial n is a repetition or alternation of the response on trial $n-1$.

There are two main reasons for investigating the generalized conditioning models. The first is that they permit a possible discrepancy between the observable experimenter-defined outcomes and the unobservable subjective reinforcing events, a distinction urged by Estes and Suppes (1959a,b). Secondly, the various possible restraints defined above on the conditioning parameters facilitate systematic and detailed study of the relationships and events that are most important in conditioning. The class G.C.M. I has been extensively studied in Suppes and Atkinson (1960). Class G.C.M. II originates with this paper, and it is intended to provide a tool for the deeper analysis of the nature of reinforcement, particularly in the direction of divorcing the definition of reinforcement from concrete single events like flashing lights and moving it toward more complex relational patterns. Basic work of this kind has already been done by Anderson and Grant (1957), Anderson (1960), and Anderson and Whalen (1960).

Method

Subjects.--The Ss were 20 students (15 female and 5 male) obtained from a School of Social Work in Liège, Belgium.

Apparatus.--The apparatus was similar to that used by Suppes and Atkinson (1960, Chp. 3).

The Ss sat at a table in front of a 40-in. wide by 50-in. high black panel, placed 20-in. from the end of the table. On the table was a square box 9-in. wide by 2-in. high terminated with a 10-in. high vertical panel. The horizontal part of the box contained two self-releasing keys mounted 6-in. apart and 9-in. from the end of the table. Upon the vertical panel were three glow panel lights. One of them, the signal light, was mounted in the center between the two keys at a height of 6-in. from them. Each of the two others, the reinforcing lights, was placed exactly above one of the keys at a height of 2-in. from it. The presentation and duration of the lights were automatically controlled by an apparatus placed in another room. On each trial the lighting of one reinforcing lamp was determined in the following manner: the last two S's responses were recorded by a relay circuit and stored; according to their particular combination the corresponding π_{ij} schedule of reinforcement written on a punched tape, was automatically read and applied.

Procedure.--The S was read the following instructions (in French): "This apparatus has three lamps and two keys. Your job is to predict, through a series of trials, which one of the two lower lamps will light. A trial goes like this: the top center lamp will light to indicate the start of the trial. As soon as it goes on you have to predict by pressing one of the two keys which one of the two lower lamps will light thereafter. You must press the key which is placed just under the lamp which you predict to light. For example, if you expect the left lamp to light, press the left key; if you expect the right lamp to light, press the right key. If you don't know what to predict, nevertheless press one key but never both. In pressing one of the keys you are doing two things: first, you indicate your prediction which is recorded, secondly you permit the lighting of one of the lights, lighting which is predetermined by a very complicated program. You must press the key until you have clearly seen which one of the lamps goes on; then you must release the key immediately (this was a requirement for the good functioning of the apparatus.) If the light above the key you pressed goes on, your prediction was correct; if the opposite light goes on, you were wrong, and should have pressed the other key.

Remember this: 1. You must press one key as soon as the top lamp lights, in any case, even if you don't know what to predict. 2. During a trial you should press only one time and only one key. Any other way will result in the recording of your prediction as a failure for that trial.

You may not be correct all the time, nevertheless try to make the best prediction you can on each trial."

For one half of the _Ss_, randomly assigned, the left response key was designated response 1 and the right, response 2; for the remaining half, vice versa. Half of the _Ss_ has response 1 reinforced on the first trial, the other half, response 2.

400 trials were run without break. On all trials the duration of the signal light was 1.5 sec.; the interval between successive signal exposures was 4 sec.; the reinforcing light immediately followed the *S*'s response with a constant delay of 0.1 sec.

The *S* was interviewed at the end of the session; the dependency of the reinforcing event on two successive responses was noticed by none.

Results and Discussion

Mean learning curves.--In Figure 1 are given the learning curves corresponding to the four pairs of responses and to the single A_1 response. Because $P(A_{1,n}A_{2,n-1})$ is approximately equal to $P(A_{2,n}A_{1,n-1})$ independently of any particular model, the A_1A_2 and A_2A_1 curves are combined in the figure. Mean proportions for successive blocks of 40 trials are given. Inspection of the figure indicates that responses are quite stable over the last four blocks, i.e., the last 160 trials.

Independently of any model it may be inferred from Figure 1 that the response pairs of *Ss* occurred with a rank ordering of frequency to be expected from the double contingent reinforcement schedule defined by (1). Thus *Ss* were most likely to be correct when responding A_1A_1, next most likely with A_2A_2, and least likely with the alternation A_1A_2 or A_2A_1. These anticipated simple qualitative effects of reinforcement are strongly supported by the data.

Stationarity of Markov process.--The test of the hypothesis that the learning process observed in this study was a stationary Markov process, implying that the transition probabilities are constant, was

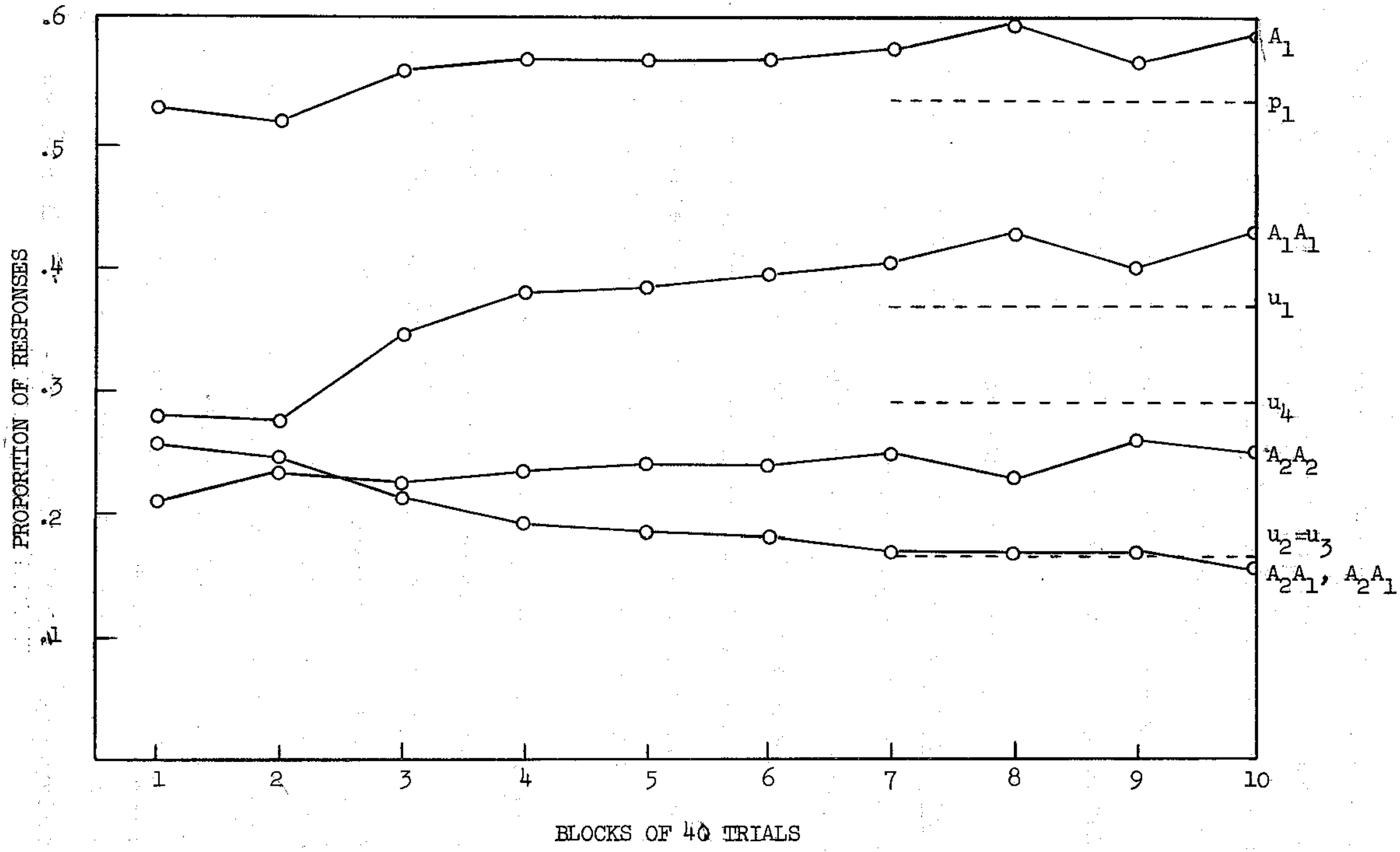

Fig. 1. Observed proportions of the response pairs and of A_1 responses in blocks of 40 trials, and asymptotic predictions from one-element θ model.

made by means of a χ^2 analysis (see Suppes and Atkinson(1960,Chp. 2) as reference for all statistical tests in this paper). The states were defined in accordance with the one-element θ model (i.e., A_1A_1, A_1A_2, etc.); the value of χ^2 computed over all blocks of 40 trials except the first two was 45.3. With $4(2-1)(8-1) = 28$ df, this value is almost significant $(.05 > P > .02)$. When the analysis was restricted to the last 4 blocks, $\chi^2 = 14.2$ with 12 df, which is not significant at the .05 level $(.30 > P > .20)$. These results support the conclusion that once the asymptotic level of response is reached the process is fairly stationary but that it is not quite so during the acquisition phase of the learning. Also these results are in good quantitative agreement with the many stationarity tests reported by Suppes and Atkinson (1960).

Another definition of the states which was considered for this experiment was the pairs of response and reinforcement on the same trial. With states defined as A_1E_1, A_1E_2, A_2E_1, A_2E_2, the obtained value of χ^2 for the last 4 blocks was 8.1, which for 12 df obviously is not significant and thus corroborates the corresponding result obtained with the former definition of state. On the other hand, with the latter state definition, the χ^2 analysis extended over all but the first two blocks of trials yielded a value of 52.5 which is significant at the .01 level. Inspection of the individual contributions of χ_i^2 to the value of χ^2 indicates that only one state, A_1E_2, seems to be responsible for the significant value; its contribution alone was 29.5.

<u>Order of Markov process</u>.--The question under consideration was: knowledge of the response history for how many previous trials would

improve the prediction of the S's behavior? Again, a χ^2 analysis was used. The first null hypothesis which was tested is that no significant improvement in prediction results from knowledge of the S's responses on the previous two trials as against only one preceding trial. The obtained values of χ^2 computed over all but the first trial block and, secondly, over the last 4 blocks were respectively 183.4 and 64.1. With $4(4-1)^2 = 36$ df these values are significant at the .01 level.

The second null hypothesis which was tested is that no significant improvement results from knowledge of responses on the previous three trials as against only two trials--second order vs. third order process. Here $\chi^2 = 25.7$ which is not nearly significant with $16(4-1)^2 = 144$ df, the analysis being made for all but the first block of 40 trials.

One-element θ model.--The result of the computation of the maximum-likelihood estimate of θ, based on the last 160 trials, was $\hat{\theta} = .476$. (For the method of derivation of this estimate from the transition matrix, see Suppes and Atkinson (1960, Chp. 2).) This value was used to compute the asymptotic response probabilities u_1, u_2, u_3, u_4, and p_1 for which the obtained values are indicated by dashed lines at the right in Figure 1. The estimate of θ was also used to compute the transition probabilities of the basic transition matrix of the model. These predictions, together with the observed transition proportions on the last 160 trials, are given in Table 1. Note that exactly four conditional predictions of $A_{1,n+1}$ determine the entire 4×4 matrix. A χ^2 goodness of fit of the predicted transition matrix to the observed transition matrix was made; $\chi^2 = 24.6$, which is highly significant ($df = 3$, $P < .001$), although on a qualitative level the predictions in Table 1 are not bad. (The relatively large χ^2 results from the large number of observations.)

Table 1

Observed and Predicted Transition Probabilities For the One-Element θ Model. Observed Values and $\hat{\theta} = .476$ Based on Last 160 Trials

$n-1, n$	Pred. $A_{1,n+1}$	Obs. $A_{1,n+1}$
A_1A_1	.71	.75
A_1A_2	.43	.51
A_2A_1	.62	.64
A_2A_2	.33	.34

These results for this model are comparable to those obtained in many other experiments in different laboratories. As usual they become much worse when we look at the finer sequential structure of the data, because $P(A_{1,n+1}|E_{1,n}A_{1,n}) = 1$, etc., i.e., when a response is reinforced it should be repeated with probability 1 in this model. As the observed transition probabilities in Table 2 below show, these probability 1 predictions were far from being verified. For examination of this finer structure we turn to the generalized conditioning models.

<u>Models of the class</u> G.C.M. I.--The observed transition probabilities $P(A_{1,n+1}|E_{j,n}A_{i,n}E_{j',n-1}A_{i',n-1})$, the observed row frequencies (number of observations), the maximum likelihood estimates of the conditioning parameters, and the χ^2 goodness of fit tests for the five models of this class defined earlier are given in Table 2. Several things are to be noted about this table. First, for completely unambiguous interpretation of the table and the five models of G.C.M. I, we define more precisely than previously, the conditioning parameters of each.

$$c_{ij} = P(A_{1,n+1}|E_{j,n}A_{i,n})$$

$$d_{ij} = P(A_{1,n+1}|E_{j,n}E_{i,n-1})$$

$$e_{ij} = P(A_{1,n+1}|A_{j,n}A_{i,n-1})$$

$$f_{j'ij} = P(A_{1,n+1}|E_{j,n}A_{i,n}E_{j',n-1})$$

$$g_{i'ij} = P(A_{1,n+1}|E_{j,n}A_{i,n}A_{i',n-1})$$

Secondly the χ^2 test also depends on the probabilities $P(A_{2,n+1}|E_{j,n}A_{i,n}E_{j',n-1}E_{i',n-1}) = 1 - P(A_{1,n+1}|E_{j,n}A_{i,n}E_{j',n-1}E_{i',n-1})$, which are not shown in the table. Thirdly, the goodness of fit is to be

Table 2

Estimated Conditioning Parameters, Observed Transition Probabilities and Goodness of Fit Tests for the Ten Models of Class G.C.M. I and Class G.C.M. II

(Computed over Last 160 Trials)

n-1 n	Row freq.	Obs. $P(A_1)$	G.C.M. I					G.C.M. II				
			c_{ij}	d_{ij}	e_{ij}	$f_{j'ij}$	$g_{i'ij}$	c_x	$d_{x'x}$	e_{xy}	f_{xy}	$g_{x'xz}$
$A_1E_1A_1E_1$	204	.843	.808	.558	.748	.841	.816	.790	.792	.839	.797	.838
$A_1E_2A_1E_1$	333	.799	.808	.612	.748	.780	.816	.790	.788	.752	.797	.775
$A_1E_1A_1E_2$	317	.669	.670	.566	.748	.633	.702	.616	.628	.576	.673	.665
$A_1E_2A_1E_2$	493	.724	.670	.627	.748	.717	.702	.616	.611	.661	.673	.681
$A_2E_1A_1E_1$	86	.837	.808	.558	.640	.841	.768	.790	.788	.839	.753	.843
$A_2E_2A_1E_1$	13	.308	.808	.612	.640	.780	.768	.790	.792	.752	.753	.333
$A_2E_1A_1E_2$	375	.603	.670	.566	.640	.633	.611	.616	.611	.576	.531	.526
$A_2E_2A_1E_2$	59	.661	.670	.627	.640	.717	.611	.616	.628	.661	.531	.552
$A_1E_1A_2E_1$	106	.509	.450	.558	.514	.399	.542	.384	.372	.339	.469	.448
$A_1E_2A_2E_1$	370	.551	.450	.612	.514	.500	.542	.384	.389	.424	.469	.474
$A_1E_1A_2E_2$	14	.643	.252	.566	.514	.303	.273	.210	.208	.248	.247	.667
$A_1E_2A_2E_2$	41	.146	.252	.627	.514	.167	.273	.210	.212	.161	.247	.157
$A_2E_1A_2E_1$	408	.370	.450	.558	.335	.399	.371	.384	.389	.339	.327	.319
$A_2E_2A_2E_1$	150	.373	.450	.612	.335	.500	.371	.384	.372	.424	.327	.345
$A_2E_1A_2E_2$	164	.274	.252	.566	.335	.303	.247	.210	.212	.248	.203	.225
$A_2E_2A_2E_2$	67	.179	.252	.627	.335	.167	.247	.210	.208	.161	.203	.162
χ^2			84.4	438.2	87.5	50.5	38.9	62.8	54.3	47.0	35.4	16.6
df			12	12	12	8	8	14	12	12	12	8

evaluated relative to the consideration of sequential dependency on responses and reinforcements for two preceding trials. The results of the order tests given above support this cut-off point, but it is to be emphasized that in all likelihood somewhat worse results would be obtained from looking at the data for three previous trials. Fourthly, the degrees of freedom for the χ^2 tests shown in the table are net degrees, i.e., the number of estimated parameters has already been subtracted.

Although the goodness of fit tests of the five models of G.C.M. I are all significant at the .001 level, there are significant qualitative differences between them. The most striking thing is that the model which fits the worst is the second one, defined by the assumption that the parameters d_{ij} depend only on the two preceding reinforcements. Considerably better predictions are made by the third model whose parameters e_{ij} depend only on the two preceding responses. It is also to be observed that the fifth model is better than the fourth, i.e., that knowledge of two preceding responses and one reinforcement is superior in predictive power to knowledge of two preceding reinforcements and one response. Moreover, in the case of the fifth model with parameters $g_{i'ij}$, if row 6 and 11 in Table 2 are omitted in the computation of χ^2 because of the small number of observations ($n = 13$ and $n = 14$ respectively), $\chi^2 = 13.77$, which with 6 df is not significant at the .02 level.

These comparative results tend to impugn the basic assumption of linear models that the probability of response is asymptotically completely determined by the sequence of reinforcing events (for statement of an exact theorem, see Estes and Suppes (1959a)). Admittedly the evidence is not decisive for we have looked at only two preceding trials. The character of our results, however, strongly suggests the importance of comparison of longer sequences of reinforcements and responses; investigators reporting on the negative recency effect found in several experiments have not made this detailed sequential comparison.

Models of the class G.C.M. II.--The maximum likelihood estimates of the conditioning parameters and the χ^2 goodness of fit tests for the five models of this class are given in Table 2. The conditioning parameters may be defined in the following manner. Let x be the variable for rewarding or punishing reinforcements, let y be the variable for anticipating an alternating or repeating reinforcement, and let z be the variable for an alternating or repeating response. (Obviously the variables x,y and z each have two possible values.) The five sets of parameters are then defined by:

$$c_x = P(A_{i,n+1} | E_{x,n} A_{i,n})$$

$$d_{x'x} = P(A_{i,n+1} | E_{x,n} A_{i,n} E_{x',n-1} A_{i',n-1})$$

$$e_{xy} = P(A_{i,n+1} | E_{x,n} A_{i,n} E_{y,n-1})$$

$$f_{xz} = P(A_{i,n+1} | E_{x,n} A_{i,n} A_{z,n-1})$$

$$g_{x'xz} = P(A_{i,n+1} | E_{x,n} A_{i,n} E_{x',n-1} A_{z,n-1}) .$$

It is to be emphasized that the parameters have the same value whether i is 1 or 2, and in the case of $d_{x'x}$ whether i' is 1 or 2.

The most impressive thing about this part of Table 2 is that with the same net number of degrees of freedom the fits of G.C.M. II models are uniformly better than those of G.C.M. I. Moreover, the assumption that the conditioning can be explained simply in terms of the last reinforcement being punishing or rewarding--two parameters c_x -- yields a better fit than does any of the G.C.M. I assumptions with four parameters (c_{ij}, d_{ij} and e_{ij}). For the model defined by the eight parameters $g_{x'xz}$ the χ^2 goodness of fit test is on the borderline of significance ($.05 > P > .02$), whereas the χ^2 for the model of G.C.M. I defined by the eight parameters $g_{i'ij}$ is highly significant.

The ordering, according to their relative goodness of fit, of the three models of G.C.M. II having four parameters each, places first the model that takes account on trial n of the rewarding or punishing character of the reinforcement and whether the response on trial n is a repeating or alternating one. In second place is the model that takes account of reward or punishment, and of anticipation of an alternating or repeating reinforcement. Similarly to G.C.M. I, in third place is the model that takes account of reward or punishment on trials n and $n-1$. What is surprising in comparing the two parameters c_x with the four parameters d_{xx} in Table 2 is how slight the differences are, which we interpret to mean the $(n-1)$st reinforcement has little direct effect on the $(n+1)$st response.

Detailed analysis of the fit of the two models--one of G.C.M. I and one of G.C.M. II--with eight parameters ($g_{i'ij}$ and $g_{x'xz}$) reveals the following interesting fact. If the data of the 16×2 transition matrix are partitioned into two subsets according to whether the last reinforcement (trial n) was punishing or rewarding, opposite results on the two subsets are obtained for the two models. For easy reference call the model with parameters $g_{i'ij}$, Ig and the other IIg. When the last reinforcement is punishing $\chi^2(\mathrm{Ig}) = 4.28$ and

χ^2(IIg) = 15.34. But when the last reinforcement is rewarding, χ^2(IIg) = 1.32 and χ^2(Ig) = 34.59. (Asymptotically the χ^2 values of the individual rows of the matrix are independent, and thus direct interpretation of the results for various subsets of rows is valid.) Sundry hypotheses concerning the conceptual difference between punishment and reward immediately suggest themselves to explain these results, but until they are shown to hold on other sets of data it would seem premature to attempt elaboration.

The uniformly better results for G.C.M. II in comparison with G.C.M. I do seem to justify extensive analysis of other data and tentatively support the conclusion that the definition of reinforcement is more aptly cast in terms of complex relational patterns than in terms of concrete single events.

Summary

This study tests several learning models for a double contingent reinforcement schedule, i.e., a schedule in which the probability of a reinforcement depends on the two immediately preceding responses. Twenty Ss were run for 400 trials each on two-response apparatus with an intertrial interval of 4 sec.

The qualitative ordering of the frequency of the response pairs followed what would be expected from consideration of the simple effects of reinforcement. In addition, the one-element θ model of statistical learning theory was found to fit the sequence of response random variables fairly well, with $\hat{\theta} = .476$. For a more detailed analysis of sequential data this model is not appropriate because it predicts that a reinforced response should be repeated with probability one.

For the more detailed analysis ten generalized conditioning one-element models were considered. The conditioning parameters of five (Class I) were defined in terms of concrete positional variables like an E_1 reinforcement or an A_1E_2 response-reinforcement pair. The parameters of the other five (Class II) were defined in terms of relational variables like a punishing or rewarding reinforcement, or a repeating or alternating response. The models of Class I were introduced earlier by Suppes and Atkinson; the models of Class II originate with this paper.

The fit to the data of the relational models (Class II) was in every case better than that of the corresponding positional model (Class I). These empirical results suggest redefining effective reinforcement in terms of relatively complex relational patterns rather than in terms of simple concrete events.

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Footnotes

1. This experiment was performed in the Laboratoire de Psychopharmacologie, Institut de Thérapeutique expérimentale, Université de Liège. This research was supported by the Rockefeller Foundation and by the Group Psychology Branch of the Office of Naval Research.

2. At the Institut de Sociologie Solvay, Universite Libré de Bruxelles.

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