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Book Reviews

Fondements d'une Théorie Positive des Choix Comportant un Risque et Critique des Postulats et Axiomes de l'Ecole Américaine. BY MAURICE ALLAIS. Paris: Imprimerie Nationale. 1955. 55 pp.

THIS MONOGRAPH REPRESENTS an expanded version of the author's article which

appeared in *Econometrica*, Volume 21, pp. 503-546. Large sections of this article are reproduced verbatim in the monograph. The author's central thesis is that a rational man need not make choices or decisions so as to maximize expected utility; in this connection he is particularly critical of the American school as exemplified in the work of Friedman, Marschak, Samuelson, and Savage. In Allais' opinion the fundamental error of this school has been its neglect of the dispersion of psychological values. Given two probability distributions on the same set of consequences or values with a common mean, the prudent man will prefer, *ceteris paribus*, the distribution with the smaller variance, and the individual who favors risk-taking will prefer that one with the larger variance, so the author claims. For him this error is symptomatic of the American tendency to ignore, and even to deplore, serious consideration of the actual psychology of risk.

More generally Allais maintains that the American school has been content to give axioms for rational behavior but not to solve the important problem of *defining* rationality. For his part he defines the rational man in two ways. Abstractly, the rational man pursues logically consistent or coherent ends, and he uses appropriate means to obtain these ends. The only three important consequences of this definition, Allais says, are the preference ordering of the field of choice, the principle of admissibility (which he calls the axiom of absolute preference), and the use of objective probabilities when available. Allais' alternative empirical definition of rationality is by ostensive reference to the actual behavior of men whom it is generally agreed are rational.

It is not possible in the space of a brief review to analyze Allais' polemic against the American school or to evaluate in detail his own positive proposals, but the reviewer will essay a few remarks. From a formal standpoint, axioms like those of Savage provide a definition of rationality, but Allais regards such axioms as only yielding a pseudo-definition (p. 31). I agree with this viewpoint, for reasons which may be made clear by referring to logic. Axioms of behavior like Savage's correspond to rules of logical inference. Now rules of inference are not themselves used to define the concept of a logically valid conclusion. Roughly speaking, a conclusion is valid if and only if the conclusion is satisfied by any possible world which satisfies the premises from which it is drawn. Given any particular set of rules of inference it is then the logician's task to show that a conclusion logically follows from given premises by use of the rules if and only if it is a valid conclusion from these premises. (To show this is not by any means mathematically trivial.) As far as I know, there have been no serious attempts to define rationality in a manner analogous to the definition of logical validity. Allais sees the need, but his own "abstract" definition quoted above clearly will not do, for it is completely lacking in definiteness or precision. Obviously his three consequences cannot be derived from it by ordinary logical or mathematical argument.

Let us take Allais' three "consequences" of ordering, admissibility, and the use of objective probabilities as axioms on a par with those of the American school. (In view of the vagueness of Allais' abstract definition from which these three principles are supposed to be inferred, treating them as axioms seems appropriate.) Then it seems to me his working theory of rationality is a retreat from rather than an advance on the work of the Americans he cites, however useful his numerous

comments and criticisms may be toward developing a realistic psychology of risk. Allais is concerned to construct examples of reasonable choice that violate principles like Savage's axiom of independence (pp. 38-40) and Samuelson's axiom of substitution (pp. 40-41). To my mind the appeal of his examples rests upon introducing particular psychological considerations or hypothetically placing individuals in very special environmental situations which are not directly pertinent to the general theory of rationality. The effect of Allais' criticisms if adopted would be to make the theory of rationality a weaker and less determinate theory of behavior than it now is in the hands of the American school. For my own part, I think that the real need is for a stronger, more determinate theory. It is not difficult to construct models of choice which satisfy all of Savage's axioms, say, but are intuitively irrational, for his axioms impose no strong restraints on the way miscellaneous prior information is assessed by the decision-maker in determining his *a priori* probability distribution on the states of nature.

Finally, a technical mathematical point of some philosophical interest may be mentioned. The author is mistaken in asserting that any of the various sets of axioms of the American school is rigorously equivalent to the Bernoullian hypothesis of expected utility maximization (p. 31). A statement of the necessary and sufficient conditions for states-of-nature models *à la* Savage to satisfy this hypothesis would require enumeration of an impossible number of isomorphism types. The various sets of axioms are sufficient but not necessary. For this reason, special structural axioms, like the one of de Finetti and Koopmans which requires the existence of an arbitrarily fine partition of the set of states of nature (and thus requires an infinite number of states of nature), are not pure axioms of rationality and are by no means implied by the Bernoullian hypothesis.

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