

MICHAEL O. RABIN. *Effective computability of winning strategies. Contributions to the theory of games, Volume III*, Annals of Mathematics studies number 39, Princeton University Press, Princeton, New Jersey, 1957, pp. 147–157.

The author proves the intuitively surprising fact that there are games with a finite length of play (but an infinite number of positions) which are games of perfect information and without chance moves, yet have no effectively computable winning strategies. The source of the surprise is that it follows easily from classical results of Zermelo and von Neumann that there must exist winning strategies for the games described.

The idea of the proof is the following. Let g be an effectively computable function from integers to integers. The game defined by g is played as follows: Player I picks an integer i ; then, knowing I's choice, II picks an integer j ; finally, knowing i and j , I picks an integer k . If $g(k) = i + j$, I wins, otherwise II wins. The theorem is then that if the range G of g is a simple set, i.e., if G is recursively enumerable and the complement of G is infinite and has no infinite recursively enumerable subset, then neither player has an effectively computable winning strategy. Now II has the winning strategies, for given any i , he must only find $f(i)$ such that $i + f(i)$ is not in G . But suppose f were effectively computable. Then $h(i) = i + f(i)$ would be also, and thus the range H of h would be recursively enumerable and infinite (since i is arbitrarily large), which contradicts the hypothesis that G is simple, for H is a subset of the complement of G .

The author also shows that if the game is played repeatedly and II always uses the same effectively computable strategy, then after a finite number of plays I can discover a winning strategy.

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