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Rules of Proportion in Architecture

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The ancient Roman architect Vitruvius begins the second chapter of the first book of his *The Ten Books on Architecture* with the statement that architecture depends on order, arrangement, eurythmy, symmetry, propriety, and economy. Although the other works on architecture from the ancient world are mostly lost, there is every reason to believe that Vitruvius was stating a commonly and widely accepted view in his emphasis on order, eurythmy, which we may think of as proportion, and symmetry. In the first chapter of Book 3, he also makes the familiar classical assertion that the principles of proportion and symmetry used in architecture are in fact derived from the symmetry to be found in the shape of the human body. He even states his conclusion this way: "Therefore, since nature has designed the human body so that its members are duly proportioned to the frame as a whole, it appears that the ancients had good reason for their rule, that in perfect buildings, the different members must be in exact symmetrical relations to the whole general scheme" [Dover edition, 1960, p. 73].

In Chapter III of Book 6, Vitruvius gives specific rules for the proportions of principal rooms; I cite three typical cases.

> In width and length, atriums are designed according to three classes. The first is laid out by dividing the length into five parts and giving three parts to the width; the second, by dividing it into three parts and assigning two parts to the width; the third, by using the width to describe a square figure with equal sides, drawing a diagonal line in this square, and giving the atrium the length of this diagonal line. [177]

> Peristyles, lying athwart, should be one third longer than they are deep, and their columns as high as the colonnades are wide. Intercolumniations of peristyles should be not less than three nor more than four times the thickness of the columns. [179]

> Dining rooms ought to be twice as long as they are wide. The height of all oblong rooms should be calculated by adding together their measured length

and width, taking one half of this total, and using the result for the height. [179]

There is, however, an important point about these proportions and the concept of symmetry that Vitruvius is using that should be made explicit. In the preceding chapter of the same book Vitruvius has this to say about symmetry.

> There is nothing to which an architect should devote more thought than to the exact proportions of his building with reference to a certain part selected as the standard. [174]

The meaning of symmetry here and elsewhere in Vitruvius is not in the sense of symmetry as used in modern mathematics and physics but in the related but different sense of using a given measure as a unit of proportion. Thus the proportional relationships between various dimensions of a room or of the house itself should satisfy simple relations of proportionality. These relations need not be those of commensurability as the passage stated above about the diagonal of the square makes clear. Simple proportions can lead immediately to quadratic equations. A famous example is the Golden Section or Golden Rectangle: the width is to the length as the length is to the sum of the width and length. Vitruvius states in various places that of course it is necessary to deviate from a single set of rules of symmetry or proportion, perhaps because of the nature of the site, or perhaps because of perceptual illusions, as in the case of oars half placed into the water. Adjustments for such perceptual phenomena has a long and important tradition in ancient architecture, and Vitruvius certainly makes a place for it. On the other hand, it is obvious that an architect was expected to have explicit principles of proportion in mind in constructing a building, and to be able to defend for a particular site and particular building, the principles of proportion he adopted. It should be noted that there has been a small industry over a good part of this century and some of the last, in attempting to impose a single theory of symmetry or proportion on a great variety of ancient buildings ranging from famous temples to ampitheatres and villas. It is fairly obvious upon examination that no single concept of proportion will account for the relationships of size to be found in the dimensions of a building or in the proportions of rooms, but it does not mean that such principles play no role.

These classical ideas of symmetry or proportion in architecture, which were at least partially derived from ancient theories of music beginning with Pythagoras, were very influential in the Renaissance, much written about and much used. Perhaps the most influential architect of the Renaissance was the sixteenth-century Italian architect, Andrea Palladio, much of whose work is still preserved and who wrote one of the most influential works in the history of architecture, *The Four Books of Architecture*, published in 1570. I use the wonderful 1737 English translation by Isaac Ware, which has been reprinted by Dover. Writing in a vein that sounds very close to that of Vitruvius, this is what Palladio has to say about the proportion of rooms in Chapter XXI of Book I. "The most beautiful and proportionable manners of rooms, and which succeed best, are seven, because they are either made round (tho' but seldom) or square, or their length will be the diagonal line of the square, or the square and a third, or of one square and a half, or of one square and two-thirds, or of two squares" [p. 27]. Palladio also gives a detailed discussion in Chapter XXIII of the same book of the height of a room given its length and width. He distinguishes whether the ceilings are vaulted or flat. His three alternative rules for vaulted ceilings use just the arithmetic, geometric, and harmonic means respectively—all familiar to the Greeks since the time of Pythagoras. One is that the height should be equal to half the sum of the width w and the length l of the room, which is the rule derived from the arithmetic means. In modern notation, but in terms of proportion

$$l - h = h - w \qquad \text{(arithmetic mean)}$$

The second "geometric" rule is that as the length of the room should stand in proportion to the height, the height of the vault stands in proportion to the width. This means that the height of the vault will be the square root of the product of the width times the length, i.e.,

$$\frac{l}{h} = \frac{h}{w} \qquad \text{(geometric mean)}$$

The "harmonic" rule for the height is slightly more complicated. It may be represented by the following equation

$$\frac{l-h}{h-w} = \frac{l}{w} \qquad \text{(harmonic mean)}$$

which can easily be solved to find h and which Palladio gives an example of. Palladio ends his discussion by saying "There are also other heights for vaults, which do not come under any rule, and are therefore left for the architect to make use of as necessity requires, and according to his own judgment." Palladio has a number of other rules of proportion for the dimensions of doors and windows, and principles for the location of doors and windows. However, it will be enough for the purposes of the discussion here to restrict ourselves to the rules and remarks cited from Vitruvius and Palladio.

There are two obvious points to be made about the rules cited from Vitruvius and Palladio, and similar ones that they give. The first is that no real justification of the rules is made. There is no extended argument in Vitruvius and Palladio as to why these particular rules are the ones that should be taken seriously, and why they have the special status they are given by the authors. It may be reasonably argued that it was precisely their immersion in the classical tradition that made no argument necessary. The theory of proportion was central to this tradition. The second comment is already to be anticipated by remarks by the architects themselves. Namely, the rules are not rules that are to be followed with precision and with algorithmic dedication. They are rules that are adjusted to particular sites and situations.

By the middle of the eighteenth century a sea change in philosophical attitudes toward beauty had taken place. Not uniformly but widespread was the advancement of a wholly subjective concept of beauty. As in other matters an early bold step was taken in this direction by Hume in his famous essay *Of The Standard Of Taste* first published in 1757. This is what he has to say:

> Beauty is no quality in things themselves: It exists merely in the mind that contemplates them; and each mind perceives a different beauty . . . . To seek the real beauty, or real deformity, is as fruitless an inquiry, as to pretend to ascertain the real sweet or real bitter.

Later he has this to say, directed even more at the classical theory of proportion.

> To check the sallies of the imagination, and to reduce every expression to geometrical truth and exactness, would be the most contrary to the laws of criticism; because it would produce a work, which by universal experience, has been found the most insipid and disagreeable.

Similar sentiments were expressed in the same year by Burke in his work *Enquiry into the Origin of our Ideas of the Sublime and Beautiful.* Burke spoke in even stronger terms than Hume. He criticized the theory of proportion as being a matter of mathematical inquiry and not at all a matter of aesthetics. He objected to the classical relation between the proportions of the human body and proportions in architecture. Even more, he poked fun at the use of this idea. But these views were not confined to British philosophy. They may also be found, although expressed in quite a different idiom, in Kant's *Critique of Judgment.* As Kant insisted, with his usual method of repetition in many different passages, back of the observation of beauty there lies no concept, for the judgment of beauty has as its source the feeling of the subject, not a concept in the object as its determining ground. Kant's analysis of the nature of beauty is notable for its endlessly repeated abstractions and its lack of detailed examples, but in matters pertinent to discussion here, he does have some particular things to say that reinforce his general view. In the following paragraph he states a general objection to taking as indisputable examples of beauty geometrically regular figures:

> Now geometrically regular figures, such as a circle, a square, a cube, etc., are commonly adduced by critics of taste as the simplest and most indisputable examples of beauty, and yet they are called regular because we can only represent them by regarding them as mere presentations of a definite concept which prescribes the rule for the figure (according to which alone it is possible). One of these two must be wrong, either that judgment of the critic which ascribes beauty to the said figures, or ours which regards purposiveness apart from a concept as requisite for beauty. (Hafner edition, 1968, p. 78)

Later he speaks even more strongly against mathematical ideas of proportionality being closely connected with the nature of beauty:

> All stiff regularity (such as approximates to mathematical regularity) has something in it repugnant to taste; for our entertainment in the contemplation

> of it lasts for no length of time, but it rather, in so far as it has not expressly in view cognition or a definite practical purpose, produces weariness. On the other hand, that with which imagination can play in an unstudied and purposive manner is always new to us, and one does not get tired of looking at it. (p. 80)

The strongly subjectivistic view of beauty that received detailed statement in the eighteenth century has continued to be part of the talk about architecture both by architects and critics alike. Unfortunately, it has encouraged a rhetoric that is naive and primitive in conceptual formulation. With some notable exceptions this is as true of what Frank Lloyd Wright has to say about "organic architecture" as it is about Robert Venturi's admonitions about complexity and contradiction in modern architecture. Fortunately, in both their cases, but especially in Wright's, their architectural practice has in its underlying design much closer affinity to the classical theory of proportion and symmetry than would appear from their own descriptions of their work. Many of Wright's most famous works, for example the Johnson Administration Building in Racine, Wisconsin, exhibit a relentless pursuit of symmetry in the sense of Vitruvius that no doubt is one of the main reasons for the impressive quality of the building.

It is certainly true that we do not necessarily expect from modern architects a clear and explicit statement of how they think about the proportions of the structures they design. This is, as has already been noted, in contrast with earlier traditions. Palladio wrote about architecture in very explicit terms and also was active as an architect himself. It is disappointing that what pronouncements we do have from the great modern architects, such as Frank Lloyd Wright, le Corbusier, Walter Gropius, and Mies van der Rohe, are maddeningly vague and general in nature instead of interesting and detailed about how particular problems are solved.

Of course, most of us tend to be intellectually put off by the bald statements about proportions to be found in classical architects, like the examples from Vitruvius or Palladio cited earlier. What they are saying is not entirely wrong or mistaken, it is just put in far too simple a way. The rules are stated categorically. Even if reservations are expressed elsewhere, there is no real defense of why these particular rules should be adopted. There is no detailed attempt to give either more fundamental principles from which they may be derived or a rich account of past experience on which they are based.

But classical or modern architects are no worse in these matters than the philosophers cited. Hume and Kant work in that great tradition of philosophical legislation without empirical representation and without concern for legislative detail. The psychologically subtle question of why proportion does appeal and has appealed so strongly in our architectural evaluations of buildings is not addressed in any serious way at all by Hume or Kant, or more generally, the philosophical traditions in which they are working.

Visual illusions. Perhaps the most disappointing aspect of developments since the time of Palladio is the absence of a rich theory of architectural illusions. Already in the design of the Parthenon, specific methods for dealing with illusions were not only evolved as a matter of experience, but given general and explicit formulation as a matter of theory. A good example of this is to be found in the following quotation from Book 3, Chapter III, of Vitruvius:

> These proportionate enlargements are made in the thickness of columns on account of the different heights to which the eye has to climb. For the eye is always in search of beauty, and if we do not gratify its desire for pleasure by a proportionate enlargement in these measures, and thus make compensation for ocular deception, a clumsy and awkward appearance will be presented to the beholder. With regard to the enlargement made at the middle of columns, which among the Greeks is called εντασις, at the end of the book a figure and calculation will be subjoined, showing how an agreeable and appropriate effect may be produced by it. (p. 86)

Detailed geometric drawings computing the enlargements to be made in the middle of columns were included in various editions of Vitruvius, and were, as indicated in the quotation, part of the text from the beginning.

Given this exemplary beginning of how theoretical corrections for perception could be implemented to improve the simple mathematical theory of proportion, the current state of developments is disappointing. It is quite true that architects are generally aware of the nature of these corrections and make many empirical adjustments in the design of current structures, but it is a reasonable thesis that the gracefulness of many classical buildings is due to the relentless application of a clear set of theoretical ideas worked out in a tradition of experience of many hundreds of years.

There is at the same time an extensive scientific theory of visual illusions that can be used to develop a more adequate theory of proportion, i.e., a theory of proportion with built-in perceptual adjustments. There is scarcely any topic in the psychology of perception that has been studied more extensively than visual illusions. The experimental and theoretical literature numbers thousands of articles and books, but excellent overall synthesis and surveys exist as well. A good example is *Seeing is Deceiving: The Psychology of Visual Illusion* by Stanley Coren and Joan Girgus (1978). The existing writing of architects and even more of architectural critics and historians does not indicate anything like a thorough and useful knowledge of this literature on visual illusions. In many ways there is an unhappy contrast to the interest in the psychology of music by musicians. The intensive work on music, which in cumulative amount does not compare to that on visual perception, is marked by close collaboration between psychologists and musicians—see, for example, *The Psychology of Music*, edited by Diana Deutsch (1982). The nature of pitch, timbre, rhythm, and harmony are not Eleusinian mysteries never to be solved, but are perceptual phenomena that are subject to thorough scientific study.

There is in the older psychological literature in this century experimental study of preferences for rectangles, triangles of a certain shape, etc. aimed at understanding in a general way what is psychologically correct about the classical theory of proportion. But this approach has not been systematically extended, so far as I know, in the past several decades. In studies of this kind or in the Greek computations of *entasis*, we should be able to come to an understanding of the contributions of proportion and symmetry to our perception of beauty, and also to understand the role of illusion as well. It will not do, with Hume and the British empiricists, to think of the mind as a *tabula rasa* fixing individually on its own conception of beauty without any attention to innate capacities of perception and how they relate to the physical world. I am not suggesting for a moment that a deepened and more sophisticated theory of proportion will encompass all that is interesting about the perception of beauty in architectural structures, but I do believe that a more thoroughly developed modern theory would be able to provide on many occasions systematic reasons why we find some buildings more pleasing to the eye than others.

What we expect of a theory of proportion as a guide to the construction of beautiful structures is in many ways not much different from what we expect of a physical theory in the design of buildings. Proper use of physical theory eliminates unsound structures and also suggests new possibilities, but physical theory does not categorically dictate how the parts should be arranged. And so it is with the theory of proportion in organizing the elements of a structure aesthetically. The aesthetic elements of a building cannot be reduced to simple formulas, and neither can the physical elements of the mechanical structure and function. Yet a building, constructed without proper engineering and understanding of the strengths and weaknesses of the technology used, is unacceptable. So should it also be with the way the elements are proportionately arranged.