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THE NONINVARIANCE OF DETERMINISTIC CAUSAL MODELS

1. INTRODUCTION

It is relatively straightforward to explain the thesis of this paper as reflected in its title and also the supporting argument for the thesis. Back of this informal and conceptually transparent argument is a substantial technical set of ideas that are explained in some detail but still not at a fully technical level in the remaining sections. The reader who is interested in understanding the general idea of the noninvariance claim I am making about deterministic causal models should be able to get the gist of the argument entirely from what is said in this introductory section.

The first point is that the results come out of ergodic theory. What ergodic theory is is explained in more detail in Sections 2 and 3, but it is easy to give a simple and familiar example of an ergodic process. Consider coin flipping. For simplicity's sake, let the coin be a fair one. Then that process is ergodic. It is ergodic for two reasons. First, it is asymptotically stationary, which means that the probability distribution of what is happening is, in the long run, unchanged over time and, secondly, it has a unique asymptotic distribution that is independent of the initial starting distribution. For example, we might have agreed always to start with a head, or always to start with a tail, but this does not affect the asymptotic distribution. A slightly more complicated example of an ergodic process is a first-order Markov process, that is, a process in which what happens on trial $n + 1$ depends on what happened on trial n, where the dependence is a probabilistic one. The same two conditions obtain, namely, asymptotic stationarity and unique asymptotic distribution of states, independent of the initial distribution. (For general processes it is standard to require, for ergodicity, not just asymptotic stationarity but stationarity for all time. This slightly stronger condition will be used in the general definition given later.)

The second concept is that of two ergodic processes being measure-theoretic isomorphic. This means that their structure of uncertainty is the

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same from an abstract standpoint, meaning that there is a one-one mapping from one process to the other, preserving the probability of each event. Remarkably, for the kinds of ergodic processes I just mentioned, for example, Bernoulli processes, as exemplified by coin flipping, or finite-order Markov processes, the sameness of uncertainty structure, that is, measure-theoretic isomorphism, holds if and only if the two processes have exactly the same entropy rate. The meaning of this remarkable result is spelled out in Section 3.

But measure-theoretic isomorphism is not enough to lead to a claim of noninvariance for deterministic causal models. To support a claim of noninvariance, we need to show that, observationally, deterministic and stochastic models of certain processes are indistinguishable. It is intuitively obvious, for example, that though we can have measure-theoretic isomorphism of a Markov process of a given entropy and a Bernoulli process with the same entropy, the two processes are easily distinguished observationally, just by a simple chi-square test of dependence of the outcome of one trial on the previous one.

To get observational equivalence, we must go on to something additional about the processes, and, for this purpose, it is sufficient to go to errors of measurement in assigning a state to the process, or, in other terms, of measuring the state at a given moment. Once only finite accuracy of measurement is granted, then another remarkable theorem coming out of ergodic theory leads to the result that two processes, one stochastic and the other deterministic, although mathematically inconsistent when applied to the same phenomena, are observationally indistinguishable. A familiar example of such processes is billiards when a convex obstacle is placed in the center of the billiard table. More about this in Section 2.

Granted the indistinguishability, the noninvariance then follows as the night the day. Although it may seem clear and as bright as sunshine that empirically real processes must be either stochastic or deterministic, but not both (the possibility of being neither is one to be considered, but not relevant here), the conclusion from indistinguishability is just the opposite. One *cannot* empirically distinguish between deterministic or stochastic models of familiar processes, many of which can be taken from standard classical mechanics, if we start at the deterministic end.

The concept of noninvariance has not really been used in this context, but it is obviously one that applies, and just the way that it applies in more familiar contexts of claims about noninvariance, as, for example, in the claim that one does not have a distinguished frame of reference, either in classical or relativistic mechanics. As another instance, all physicists accept that classical simultaneity of distant events is not an invariant notion in special relativity. We certainly do in everyday life, and physicists everywhere do, use the classical notion of simultaneity in their laboratories and in relation to many experiments, but this does not change the fundamental fact about simultaneity's noninvariance in the more accurate theory of special relativity.

The parallel to the use of deterministic models is in my view very close. We use, and will continue to use, in useful fashion, deterministic causal models. But this does not mean that they are necessarily part of the fundamental furniture of the universe. We use them because they are a good computational approximation. I particularly want to stress the computational aspects. The glory of nineteenth-century physics is the wonderful development of computational methods in physics within the framework of deterministic models. That is an accomplishment that will stand forever, capped above all by Maxwell's equations and the theory that flows from those equations. But probably no physicist today considers Maxwell's equations as a description of physical reality valid "all the way down", that is, to measurements that are much smaller than a micrometer. In fact, violations can probably be established already within the range of a micrometer, certainly within the range of a nanometer, the unit of measurement we use for light.

It might be claimed that ergodic models, particularly because they are stationary models, provide a very specialized framework scientifically, for discussing most phenomena and are, therefore, not really applicable. But the stationarity is not really critical to the argument. There are various ways of getting around it. Undoubtedly, the open contexts in which most empirical phenomena take place more than offset the problems of not having stationarity available and give us more than adequate room for the kind of uncertainty structure central to the detailed argument that I am giving. It is just that we cannot carry out those detailed arguments for a passing cloud, a bird in flight, a child running or any other familiar phenomena. The detailed analysis is much too complicated, but if we could carry it out my claim about noninvariance of deterministic models would, I am sure, hold. At the very least, I am skeptical for many reasons that challenging the use of approximate stationarity will open up a clear road to determinism.

The point, and the only point, of this article is that to affirm that the world is deterministic must be taken to be a transcendental metaphysical idea for a broad class of theories and their models, not verifiable within any broad framework of data. A philosophical corollary of this argument about determinism is that much too much is made of the problem of determinism in the discussion of free will. It is the activity of goal setting and the adoption of means to achieve goals, the characteristic activities of organisms not caught in a deterministic web, that are the marks of free will. And it is this process – goal setting and the implementation of means to achieve the goals – that is empirical in character, not the universal determinism so often assumed to be the nemesis of free will and the concepts that flow from it. (For more detailed development of this latter point, see Suppes (1993, 1994, 1995).)

2. PHOTONS AS BILLIARDS

The mechanical example on which I shall concentrate has a long philosophical history, namely, the case of billiards, but I want to change that example in two ways. First, I will think of photons as billiards and, secondly, and more importantly, will introduce a convex obstacle in the middle of the billiard table, which, as we shall see, plays havoc with the ordinary ideas of billiards being a regular, smooth, easily understood deterministic mechanical system.

Keeping the convex obstacle in mind, we can move to the study of photons as particles executing ergodic motions. (A formal characterization of ergodic is given in the next section.) The analysis in this section is drawn from Suppes and de Barros (1996). Let us begin with a rectangular box that has reflecting sides. We assume the classical law of reflection, that is, the angle of reflection equals the angle of incidence. An ideal laser could execute the periodic motion shown in Figure 1 in such a box. In fact, in classical mechanics it is always in terms of such motions that billiards are used as standard examples of mechanical systems. It is only in the modern study of billiards that matters become much more complicated. Intuitively it is easy to describe how to get such additional complication. We add a convex obstacle to the rectangular box with reflecting sides as shown in Figure 2.

Now the path of the photon emitted by an ideal laser executes the motion of a photon as a Sinai billiard. (The study of billiards with a convex object in the middle of the table has been especially studied by the Russian mathematician Y. G. Sinai, and, consequently, such a billiard example is called a Sinai billiard.) Sinai and other investigators have studied very thoroughly the mechanical motion of a point particle in such a rectangular box with a convex obstacle and with the collisions of the particle observing the classical law of reflection as well as the law of perfect elasticity. In the case of photons we drop the concept of elasticity but have the reflection take place without loss of energy.

The motion of the photon billiard in Figure 1 is the classic one of periodic motion. That of Figure 2 is something new and not a topic early in

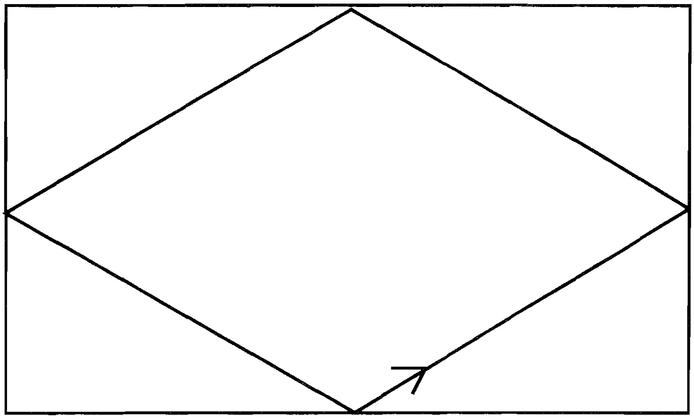

Figure 1. Photon billiard with a periodic motion

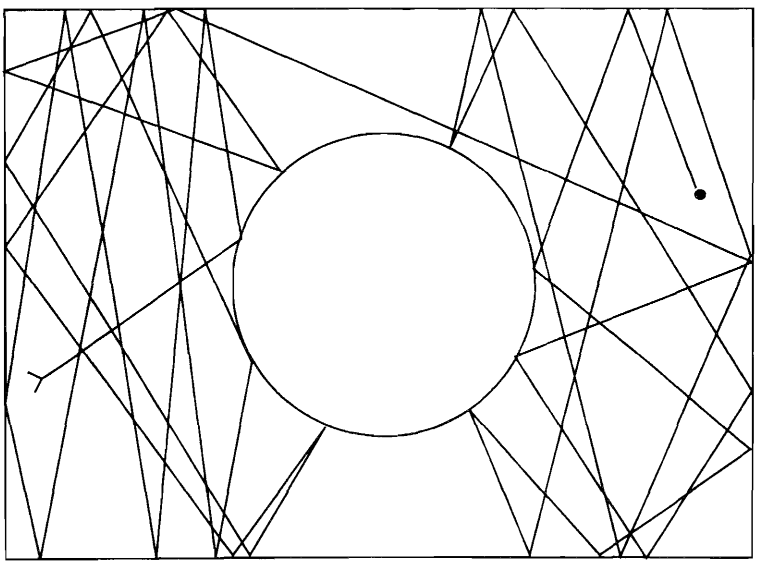

Figure 2. Photon billiard motion with a convex obstacle.

the history of mechanics. A photon billiard motion with a convex object intuitively has the property that the trajectory of the billiard extended over time will fill all of the space. A second way of expressing what the kind of motion to be seen in the billiard is is often put this way, which will receive an even clearer mathematical definition in the next section. In a mechanical process that is ergodic, the time averages equal the space averages. What this cryptic saying means is that if we follow a single particle for an extremely long time, then, if the physical process is ergodic, all behavior we have obtained by looking at the averages over ensembles or space of a given property, will be the same as the time averages. For example, to take something too simple, but vivid enough to make the concept clear, suppose we have particles moving in a box and we have a round dot painted on one of the walls. Then we can start a large number of particles moving in the box and we can compute the expectation of the number of times particles will hit the black dot, in one hour, say. We get a similar result by observing a single particle and computing its expectation of hitting the dot how many times in n hours, if the first example contained n particles. If the process is ergodic, these two expectations should be the same. So the most striking idea of ergodicity is that a single particle, observed long enough in time, will exhibit all the behavior of an arbitrarily large collection of particles of the same type and in the same environment.

3. ISOMORPHISM AND INDISTINGUISHABILITY OF ERGODIC PROCESSES

A stochastic process $\mathcal{X}$ is an indexed family $\{\mathbb{X}_n\}$ of random variables. The index, discrete or continuous, is usually interpreted as time, and so it will be here. For simplicity and without any real conceptual loss, I consider only the discrete case with $n = 1, 2, 3, \ldots$, although some remarks will concern the doubly infinite case, $n = \ldots -2, -1, 0, 1, 2, \ldots$. The usual assumption about the collection of joint probability distributions of any finite subsequence of the random variables being consistent is made. (This section follows Suppes (1996, 1997); for more general analysis see Ornstein and Weiss (1991).)

The appropriate concept of entropy for a stochastic process $\mathcal{X}$ is that of *entropy rate* $H(\mathcal{X})$ defined as follows

$$H(\mathcal{X}) = \lim_{n\to\infty} \frac{1}{n} H(\mathbb{X}_1, \ldots, \mathbb{X}_n),$$

provided the limit exists. (Notice that $H(\mathbb{X}_1, \ldots, \mathbb{X}_n)$ is just the entropy of the first n random variables. We convert to a rate by dividing by n.)

A (discrete, finite) Bernoulli process is a stochastic process that is a sequence $\mathbb{X}_1, \mathbb{X}_2, \ldots$, or possibly a doubly infinite sequence, with the $\mathbb{X}_n$'s independent and identically distributed random variables with a fixed finite range of values. It is easy to show that such a Bernoulli process $\mathcal{X}$ has entropy rate

$$H(\mathcal{X}) = \lim_{n\to\infty} \frac{H(\mathbb{X}_1, \mathbb{X}_2, \ldots, \mathbb{X}_n)}{n} = \frac{nH(\mathbb{X}_1)}{n} = H(\mathbb{X}_1)$$

$$= -\sum p_i \log p_i.$$

where p_i is the probability of outcome i, independent of the trial n. So, in the familiar process of flipping a fair coin, let 1 = heads and 2 = tails. Then $p_1 = p_2 = \frac{1}{2}$.

More generally, for a stationary process the entropy rate, as defined above, can be shown to be equal to the conditional entropy rate, defined as

$$H'(\mathcal{X}) = \lim_{n\to\infty} H(\mathbf{X_n}|\mathbf{X_{n-1}}, \ldots, \mathbf{X_1}),$$

provided the limit exists, which it does for stationary processes. (A process is *stationary* if the probability distribution of any k-tuple $(\mathbf{X}_{n+1}, \ldots, \mathbf{X}_{n+k})$ is independent of n, intuitively, independent of time.) For a (first-order) stationary Markov process,

$$\begin{aligned} H'(\mathcal{X}) &= \lim H(\mathbf{X_n}|\mathbf{X_{n-1}}, \ldots, \mathbf{X_1}) \\ &= H(\mathbf{X_2}|\mathbf{X_1}) \\ &= -\sum_x p(x) \sum_y p(y|x) \log p(y|x) \end{aligned}$$

where $p(y|x)$ is the transition probability of going from state x to state y on any trial.

I next consider a standard probability space $(\Omega, \Im, P)$, where it is understood that $\Im$ is a σ-algebra of subsets of Ω and P is a σ-additive probability measure on $\Im$. Let T be a mapping from Ω to Ω. We say that T is *measurable* if and only if whenever $A \in \Im$ then $T^{-1}A = \{\omega\colon T\omega \in A\} \in \Im$, and even more important, T is *measure preserving* if and only if $P(T^{-1}A) = P(A)$. The mapping T is *invertible* if the following three conditions hold: (i) T is $1-1$, (ii) $T\Omega = \Omega$, and (iii) if $A \in \Im$ then $TA = \{T\omega\colon \omega \in A\} \in \Im$. In the application we are interested in, each ω in Ω is a doubly infinite sequence and T is the *right-shift* such that if for all n, $\omega_n = \omega'_{n+1}$ then $T(\omega) = \omega'$. Intuitively this property corresponds to stationarity of the process – a time shift does not affect the probability laws of the process, and we can then use T to describe orbits or sample paths in Ω.

We now characterize isomorphism of two probability spaces on each of which there is given a measure-preserving transformation, whose domain and range need only be subsets of measure one, to avoid uninteresting complications with sets of measure zero that are subsets of Ω or Ω'. Thus we say $(\Omega, \Im, P, T)$ is *isomorphic in the measure-theoretic sense* to $(\Omega', \Im', P', T')$ if and only if there exists a function $\varphi\colon \Omega_0 \to \Omega'_0$ where $\Omega_0 \in \Im$, $\Omega'_0 \in \Im'$, $P(\Omega_0) = P'(\Omega'_0) = 1$, and φ satisfies the following conditions:

(i) φ is $1 - 1$,

(ii) If $A \subset \Omega_0$ and $A' = \varphi A$ then $A \in \mathfrak{I}$ iff $A' \in \mathfrak{I}'$, and if $A \in \mathfrak{I}$

$$P(A) = P'(A'),$$

(iii) $T\Omega_0 \subseteq \Omega_0$ and $T'\Omega_0' \subseteq \Omega_0'$,

(iv) For any ω in Ω_0

$$\varphi(T\omega) = T'\varphi(\omega).$$

I emphasize that the isomorphism in the measure-theoretic sense of two processes seems at the right level of abstraction. The isomorphism expresses that the two structures have the same degree of uncertainty, even though they differ considerably in other characteristics.

The isomorphism in a measure-theoretic sense of two stationary stochastic processes provides the important step of giving us a meaningful basis in terms of uncertainty for judging their equivalence. Note why this is so. The φ function mapping one process into another is measure-preserving, so there is a structural isomorphism between corresponding events of the two processes such that they have the same probability. It is precisely the fact that the mapping carries events into events of the same probability that supports the claim that isomorphism in the measure-theoretic sense represents equivalence of uncertainty.

On the other hand, it is equally important to note that isomorphism in the measure-theoretic sense of two stochastic processess only means isomorphism in the structure of uncertainty, as I have called it. Such isomorphism does not imply observational equivalence, nor would we want it to. For example, a Bernoulli process and a Markov process with strong dependence from one trial to the next can be isomorphic in the measure-theoretic sense but easily distinguishable by a chi-square test for dependence. What we want to be able to say about these two processes is that they are equivalent in terms of uncertainty, but clearly different in other respects.

To show how recent fundamental results are about the relation between entropy rate and measure-theoretic isomorphism, I note that it was an open question in the 1950s whether the two finite-state discrete Bernoulli processes $B(\frac{1}{2}, \frac{1}{2})$ and $B(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ are isomorphic. (The notation here should be clear; $B(\frac{1}{2}, \frac{1}{2})$ means that the probability for the Bernoulli process with two outcomes on each trial is that for each trial the probability of one alternative is $\frac{1}{2}$ and of the other $\frac{1}{2}$.) The following theorem clarified the situation.

THEOREM 1 (Kolmogorov 1958, 1959 and Sinai 1959). If two finite-state, discrete Bernoulli or Markov processes have different entropies, then they are not isomorphic in the measure-theoretic sense.

Then the question became whether or not entropy is a complete invariant for measure-theoretic isomorphism. The following theorem was proved a few years later by Ornstein.

THEOREM 2 (Ornstein 1970). If two finite-state, discrete Bernoulli processes have the same entropy rate then they are isomorphic in the measure-theoretic sense.

This result was then soon easily extended.

THEOREM 3 (Adler, Shields and Smorodinsky 1972). Any two irreducible, stationary, finite-state, discrete Markov processes are isomorphic in the measure-theoretic sense if and only if they have the same periodicity and the same entropy.

We then obtain:

COROLLARY 1. An irreducible, stationary, finite-state, discrete Markov process is isomorphic in the measure-theoretic sense to a finite-state, discrete Bernoulli process of the same entropy rate if and only if the Markov process is aperiodic.

I return now to photons as billiards and apply the concept of measure-theoretic isomorphism. To keep things in the context of finite-state discrete processes, we can form a finite partition of the free surface on the billiard table, as shown in Figure 3.

This constitutes a finite partition of the space of possible trajectories for the photon or billiard and we correspondingly make time discrete in terms of movement from one element of the partition to another. With these constructive approximations, the following theorem has been proved:

THEOREM 4 (Gallavotti and Ornstein 1974). With the discrete approximation of the continuous flows just described above, the discrete deterministic model of the photon or billiard is isomorphic in the measure-theoretic sense to a finite-state discrete Bernoulli process model of the motion of the photon or billiard.

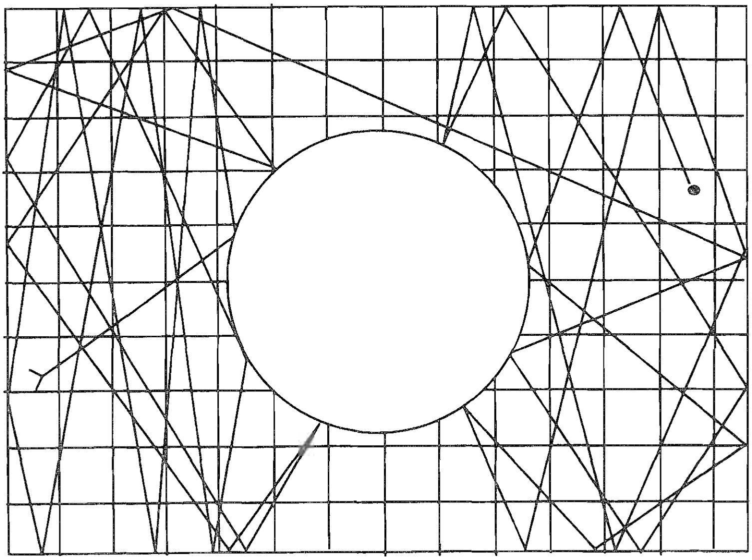

Figure 3. Finite partition of the billiard table with convex obstacles.

It should be noted that instead of this theorem we could have stated a theorem for continuous time and such results are to be found in the paper by Gallavotti and Ornstein. What the Gallavotti and Ornstein theorem shows is that the discrete mechanics of billiard balls is in the measure-theoretic sense isomorphic to a discrete Bernoulli analysis of the same phenomena. However, it is to be emphasized that in order to claim that intuitively the two kinds of analysis are indistinguishable from observation we need stricter concepts.

However, before going on to questions of indistinguishability, we need to return to a more formal discussion of when a process is ergodic. In the intuitive discussion of the Sinai billiards, I stated the familiar maxim that, for ergodic processes, the space averages equal the time averages, or, put another way, ensemble averages are the same as the time averages. It will be useful to take a very simple example of an obvious ergodic process, but one lasting for only 8 trials, and consider the meaning of this in a completely explicit way. Consider on the one hand, flipping a fair coin 8 times, i.e., where we have 8 trials. On the other hand, consider flipping 8 fair coins once. We get a space Ω and a probability measure on that space in each of these cases. It is intuitively obvious that one space corresponds to an ensemble cross-section, namely, what happens on a given trial, and the other to 8 flips of the same coin. It is easy to establish the mapping ϕ between them that makes them measure-theoretic isomorphic. (I am deliberately omitting the transformation T referred to in the earlier technical discussion.) Consider the event in the case of the ensemble of a head for the third coin and a tail for the seventh coin. Then this event would map in a natural way into the time sequence of a head on the third trial and a tail on the seventh trial for the single coin. Now, in order for things to work smoothly, to introduce the transformation T, we of course need to have a stationary process, tending at least forward infinitely, and, preferably backward as well. It is obvious how we could continue this kind of example by enlarging our two processes, but I shall not do so, but move on to more systematic considerations, namely to the standard definition of a process being ergodic. When we talk about such a process, we have in mind a probability space and a family of random variables defined on that space.

First, we define a set A of the probability space to be *invariant* under the transformation T in case $TA = A$, that is, T applied to all elements of A simply reproduces A. (Note that I am restricting the definition to the case of T being invertible for simplicity.) Second, we define T to be *ergodic* if each invariant set A is trivial in the sense of having measure either 0 or 1, i.e., $P(A) = 0$ or $P(A) = 1$. Third, we are now in a position to state the fundamental ergodic theorem for discrete processes, the theorem that expresses that time averages equal ensemble averages. First, we introduce the usual notation I_A for the indicator function of the set A, that is, for each ω in Ω

$$I_A(\omega) = \begin{cases} 1 & \text{if } \omega \text{ is in } A \\ 0 & \text{if } \omega \text{ is not in } A \end{cases}$$

The fundamental theorem then says that for every set A, that is, every event A, almost every ω enters A with asymptotic relative frequency $P(A)$. More explicitly,

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=0}^{n-1} I_A(T^k\omega) = P(A).$$

(The "almost every" means with the possible exception of a set of ω's of measure 0.) Note again this theorem holds for an individual sample path ω.

Keeping in mind this theorem and the definition of ergodic, Sinai (1976) proved the following difficult theorem for the continuous case of the motion of a Sinai billiard.

THEOREM 5. The motion of a Sinai billiard, as shown in Figure 2, is ergodic.

3.1. *Indistinguishability and α-Congruence*

To obtain a stricter sense of isomorphism it is natural to impose a geometric conditon, especially for a wide variety of physical examples of ergodic systems. Here we follow Ornstein and Weiss (1991). Let $\alpha > 0$ and let $\chi = (\Omega, \Im, P, T)$ and $\chi' = (\Omega', \Im', P', T')$ be two spaces isomorphic under φ in the measure-theoretic sense. Then χ and χ' are *α-congruent* if and only if there is a function g from Ω to a metric space (with d the metric) and a function g' from Ω' to the same metric space such that for any ω in Ω, $d(g(\omega), g'(\varphi(\omega))) < \alpha$ except for a set of measure $< \alpha$.

Intuitively the parameter α reflects our inability to measure physical quantities, including geometric ones, with infinite accuracy. What is significant is that α-congruence for small α, can be proved for Sinai billiards, and thus photons in a Sinai billiard box. And when α is chosen at the finite limit of our measurement accuracy, the Newtonian mechanical and a Markov process model of a Sinai billiard are observationally indistinguishable, as they are α-congruent.

Stated informally, we then have the fundamental result.

THEOREM 6 (Ornstein and Weiss 1991). Using the discrete approximation described just before Theorem 4, the discrete deterministic model of the photon or billiard is observationally indistinguishable from a finite-state discrete Markov model of the motion of the photon or billiard.

Extension of this theorem to continuous flows is to be found in Ornstein and Weiss (1991).

4. NONINVARIANCE

The principal noninvariance argument concerning deterministic causal models can be given in a formal schematic way quite simply. Let $\mathcal{M}$ be the class of ergodic mechanical models such as billiards with a convex obstacle. There are now in the literature a number of examples of such mechanical systems (Ornstein and Weiss (1991) and Sinai (1976)). Restricting ourselves to classical mechanics, we can extend $\mathcal{M}$ by using the set $\mathcal{G}$ of all Galilean transformations to other inertial frames of reference. So if M is in $\mathcal{M}$ and g is in $\mathcal{G}$ then $g(M)$ is in $\mathcal{M}$. (The necessarily somewhat elaborate mathematical characterization of $\mathcal{G}$ can be found in McKinsey and Suppes (1953, 1955); also included in these references is the extension to generalized Galilean transformations, which include changes of measurement scales as well as going from one inertial frame of reference to another.)

Notice two points about M and $g(M)$. First, empirical evidence e about the behavior of any mechanical model M applies equally well, interpreted as $g(e)$ to $g(M)$. Moreover, critical properties of M, such as absolute magnitude of acceleration a, are invariant under g, i.e., if M has scalar acceleration a, $g(M)$ has a also. (The direction vector of acceleration a is, of course, covariant rather than invariant.)

Now, in the light of Ornstein and Weiss' theorem (Theorem 6 of the last section), let $\mathcal{S}$ be the set of stochastic processes observationally equivalent to some mechanical model M in $\mathcal{M}$. Let φ be a mapping of $\mathcal{M}$ onto $\mathcal{S}$ such that M and $\varphi(M)$ are observationally equivalent. Then the property of being deterministic is not an invariant property of observationally equivalent models of ergodic mechanical phenomena under the realistic assumption that measurements of continuous quantities can have only finite accuracy, i.e., be accurate only to a fixed finite number of digits for a fixed scale of measurement.

I come now to the second point. Someone might object to the above argument on the grounds that the concept of being invariant or not was not meant to apply to the relationship between the models of two theories that are clearly mathematically inconsistent, as is the case for the theory of classical mechanics and the theory of the associated stochastic processes in $\mathcal{S}$. But what is not recognized often enough is that, literally speaking, the same formal objection applies to the empirical statements made with respect to two different inertial frames of reference. The natural example, astounding from the standpoint of Aristotelian physics and naive commonsense is this. The assertion that a mechanical system observer is at rest with respect to one frame of reference is, of course, phenomenologically inconsistent with another Galilean observer asserting that the same mechanical system is moving with a positive uniform velocity. But classical physics insists on their dynamical equivalence. Being at rest is not a concept that has an absolute meaning in classical physics, only the relative concept of rest or motion.

We could not get along without the concept of an object being at rest with respect to our bodies or other bodies. Think of it being arbitrary whether or not the furniture in any room you occupied would be moving about from one moment to another. This can, of course, be true in a space ship if the furniture is not bolted down. But bolting is just what is done to replace the force of gravity.

The same can be said for our everyday use of deterministic ideas, very useful and practical when applied in a restricted and properly circumscribed way, but intellectually dangerous and misleading when applied too broadly, as in general arguments about the causal structure of the universe or the nature of free will.

A response of a different sort to the arguments I have been giving is sometimes heard. It goes like this. "Well, yes, if we mix physical theory and the empirical problem of methods of measurement with the actual accuracy of the results, it is not surprising a theorem like Ornstein's can hold for ergodic mechanical systems, but this is mixing apples and oranges improperly. It has been no part of the grand classical tradition of theoretical mechanics to consider errors of measurement. Such errors are a problem for applied mechanics and engineering."

My rejoinder is twofold, partly classical and partly quantum mechanical. Concerning the classical part, it is, I claim, a serious misreading of the history of classical mechanics to hold that the central concerns have been purely theoretical. Three of the greatest contributors to mechanics from the seventeenth to the nineteenth centuries were Newton, Laplace and Gauss, all three of whom were deeply committed to observational astronomy and the development of the theory of errors. These several concerns in the hands of these three extraordinary scientists led to advances in astronomy that would not have been possible without the detailed theory of error they primarily developed. As Gauss says in his memoir on the theory of least squares (1821), "This problem [of minimizing the estimated error] is certainly the most important which the application of mathematics to natural philosophy presents."

It is in fact the glory of astronomy, as the earliest developed branch of mechanics, to have led the way from ancient times by being the first science to engage in the sustained application of mathematics to observational data. The intricacies of this history are well illustrated in Laplace's qualitative but detailed record in Chapter IX of his *Philosophical Essay on Probabilities* (1820/1951). This chapter emphasizes the continuing interplay between improving the limits of measurement error and the potential for new and better theoretical calculations about the mass of Saturn and of Jupiter, the oblateness of the earth and lunar motion, the motion of the sun and the secular equation of the moon, the irregularities in the motion of Jupiter and Saturn, and still other examples I shall not mention.

A skeptical and historically minded philosopher might still object to the argument about the centrality of error computations to the continued theoretical development of celestial mechanics along the following lines. "Well, if error is so central, why did these scientists of unparalleled talent not directly introduce errors into the fundamental theory of mechanics and formulate the theory in terms of stochastic processes?" The answer is that this was too difficult, and even the basic tools for solving stochastic differential equations were not available until the middle of this century in the form of the Ito calculus. Moreover, even with this powerful development the difficulties of a continuous stochastic formulation remain formidable.

Much more can be said on this topic, but I turn now from classical physics to quantum mechanics. Here the mixing of theory and measurement is now viewed as an essential aspect of the theory, both in terms of the Heisenberg inequality for the product of the variances of two conjugate variables such as position and momentum, and in terms of what is called the measurement problem in quantum mechanics. So the mixing of theory and errors of measurement, as is done in Theorem 6, is not special to Ornstein and Weiss' equivalence theorem, but is to be found in many places.

One final coda on this point about error. This is not really in many respects the best way to formulate the central problem of only finite accuracy. The variability in the surrounding environment and in the activities of physical measurement itself reflect a variability in nature that is in fact uneliminable. This is the way the world is and it is a deep-seated mistake to think otherwise. So the choice between deterministic or stochastic models of ergodic systems remains with us, and we are free to choose which we prefer for historical, metaphysical or perhaps computational reasons.

4.1. *Can Determinism Be Saved?*

It has been my intention to make the case for a stochastic model of any ergodic mechanical system. This seems to bias the metaphysical choice toward indeterminism and stochastic processes.

Given a stochastic process – forgetting for the moment about ergodic mechanical systems, can we save determinism for those with that kind of metaphysical predilection? The answer is affirmative and detailed for a surprisingly wide variety of cases, such as coin-flipping, discrete or continuous Markov processes, etc. To keep the notation simple I consider only a finite sequence of random variables $\mathbf{X_1}, \ldots, \mathbf{X_n}$.

In the language of quantum mechanics the deterministic common cause λ of the following theorem would be referred to as a *hidden variable*, but the theorem in itself has no direct connection with quantum mechanics.

THEOREM 7 (Suppes and Zanotti 1981; Holland and Rosenbaum 1986). Let n random variables $\mathbf{X_1}, \ldots, \mathbf{X_n}$, finite or continuous, be given. Then there exists a common cause $\boldsymbol{\lambda}$ such that there is a joint probability distribution F of $(\mathbf{X}_1, \ldots, \mathbf{X}_n, \boldsymbol{\lambda})$ with the properties

(i) $F(x_1, \ldots, x_n \mid \lambda) = P(\mathbf{X}_1 \leq x_1, \ldots, \mathbf{X}_n \leq x_n \mid \boldsymbol{\lambda} = \lambda)$

(ii) Conditional independence holds, i.e., for all $x_1, \ldots, x_n, \lambda$,

$$F(x_1, \ldots, x_n \mid \lambda) = \prod_{j=1}^{n} F_j(x_j \mid \lambda),$$

if and only if there is a joint probability distribution of $\mathbf{X}_1, \ldots, \mathbf{X}_n$. Moreover, $\boldsymbol{\lambda}$ may be constructed so as to be deterministic, i.e., the conditional variance given $\boldsymbol{\lambda}$ of each $\mathbf{X}_i$ is zero.

To be completely explicit in the notation

(1) $$F_j(x_j|\lambda) = P(\mathbf{X}_j \leq x_j \mid \boldsymbol{\lambda} = \lambda).$$

4.2. *Idea of the Proof*

Consider, as a simple example, three ± 1 random variables $\mathbf{X}$, $\mathbf{Y}$ and $\mathbf{Z}$. There are 8 possible joint outcomes $(\pm 1, \pm 1, \pm 1)$. Let p_{ijk} be the probability of outcome (i, j, k). Assign this probability to the value λ_{ijk} of the hidden variable $\boldsymbol{\lambda}$ we construct. Then the probability of the quadruple (i, j, k, λ_{ijk}) is just p_{ijk} and the conditional probabilities are deterministic, i.e.,

$$P(\mathbf{X} = i, \mathbf{Y} = j, \mathbf{Z} = k \mid \lambda_{ijk}) = 1,$$

and factorization is immediate, i.e.,

$$P(\mathbf{X} = i, \mathbf{Y} = j, \mathbf{Z} = k \mid \lambda_{ijk})$$
$$= P(\mathbf{X} = i \mid \lambda_{ijk}) P(\mathbf{Y} = j \mid \lambda_{ijk}) P(\mathbf{Z} = k \mid \lambda_{ijk}).$$

Extending this line of argument to the general case proves the joint probability distribution of the observables is sufficient for existence of the factoring common cause. From the formulation of Theorem 7 necessity is obvious, since the joint distribution of $(\mathbf{X}_1, \ldots, \mathbf{X}_n)$ is a marginal distribution of the larger distribution $(\mathbf{X}_1, \ldots, \mathbf{X}_n, \boldsymbol{\lambda})$.

It is obvious that the construction of $\boldsymbol{\lambda}$ is purely mathematical. It has in itself no physical content. The absence of physical content can be regarded as desirable in a purely metaphysical view of determinism. But certainly those philosophers who want to bedevil advocates of the empirical reality of free will with the threat of universal determinism, will not be satisfied with this theorem. Others, more realistic about what could ever be hoped for in proof of a universal thesis of determinism, may feel metaphysically satisfied. In any case, the theorem does not affect the thesis of noninvariance that has been my focus, for this is a thesis about empirical physics, idealized to be sure, but idealized in the style of theoretical physics not metaphysics.

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