Variable-free Semantics for Stochastic Processes: A Preliminary Report

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1 Background

There has been much early work on variable-free semantics and I will not attempt to review that here, but only mention the important systematic recent work of Tarski and Givant [7]. As is familiar to many people, Tarski and Givant show that a variable-free formalization of set theory can express a large part of standard mathematics, a much larger part than almost anybody would expect to be the case. They also establish the limitations of the system where the expressive power breaks down, but I shall not expand on the details here. I use this as a reference because of its encouraging character.

Concerning my own prior work on variable-free semantics, the work on logical inference in English [10] is the most relevant, but I shall not review it here. We can expect to do a great deal with variable-free semantics, even in the context of mathematical objects as complicated as stochastic processes. For a full treatment of stochastic processes, we would also need a much more explicit lexical semantics than I have given in any previous work, or than has in general been given by anyone else dealing with variable-free semantics for ordinary English or some other natural language. Following a hallowed tradition, I will omit the discussion of lexical semantics. I just want to remark that in recent detailed work on robotics and on physics word problems, we have found that it is absolutely essential to enter into the lexical semantics in some detail.

2 New Concepts Needed

To extend to the kind of temporal processing and consideration of events required for stochastic processes, I introduce several new concepts for the analysis of ordinary language statements about aspects of stochastic processes. Remember, this is not about the theory of stochastic processes, but the way in which we use, in an easy and informal way in natural language, ideas that are naturally put from a theoretical standpoint in the framework of stochastic processes. This means all kinds of ordinary statements about uncertain events, about chance occurrences or accidental matters from the past, present or future.

The main three concepts I want to use are the following:

Events. I shall use capital letters of the Roman alphabet to denote events. I shall especially use H and T for heads and tails.

Temporal ordering of events. I shall use ordinary numerical inequalities for the temporal ordering of events to distinguish this relation from the ordering relation of qualitative probability, even though I shall not assume a numerical index of time. This temporal ordering of events will apply whether the events are discrete or continuous and whether time is treated as discrete or continuous, a matter that usually doesn't arise in any very decisive way in ordinary use of event language.

Qualitative probability, which symbolically is usually written as $H \succ T$ and which is read as "Heads is more probable than tails."

We need still more the standard qualitative notion of conditioning, which we write $H \mid T \succ H \mid H$. A head, given a tail, is more likely than a head given a preceding head. Notice in this case, just for variation, instead of using the strict qualitative probability inequality, I could have used the weak one, $\succeq$. When I talk about heads and tails, I am, of course, referring to coin flipping. These are only examples. What I am doing in terms of notation is in no sense restricted to heads and tails.

3 Concepts for Stochastic Processes

There are a series of concepts about stochastic processes that we will need. Some of them are rather technical and I will not say much about them, but they are important as background for what follows.

1. *Stationary vs. nonstationary processes.* Intuitively, a stochastic process is stationary when a translation in time makes no difference to the process. In other words, the general character of the behavior of the process is independent of any particular time. So, for example, in coin flipping the probability of a head in an ordinary Bernoulli process is the same on two different trials, no matter how separated in time they may be. Of course, it is important to emphasize that this does not mean that we would have the same outcome on these two trials, it is just that the process is stationary in terms of its probability setup.

Nonstationary processes do not have this property. Among the most familiar nonstationary processes are any stochastic processes for learning. We expect probabilities of correct responses to change under a regime of learning. Our own daily experience can be broadly separated into stationary and nonstationary processes. Much of our daily routine we think of intuitively as being a stationary process. Children going to school every day at about the same time constitute a standard example. On the other hand, all kinds of special events that occur do not constitute anything like a stationary process.

2. *Ergodic vs. nonergodic processes.* Roughly speaking, an ergodic process is one in which we have no dependence on initial conditions as they become far removed from current events and in which we have a unique limiting distribution as time goes on. To put the matter exactly, for discrete time finite-state Markov chains, such chains are ergodic if and only if there exists a unique asymptotic probability distribution of the

states independent of the initial distribution. Not all processes are ergodic. Certainly, it is one of the strong claims of Freudian psychology that childhood experiences do not geometrically fade away into the past, that is, with a geometric distribution, but rather, remain, if not in full force, in some strength many years later. It is a conception of experience that gives it a nonergodic property.

3. *Independence vs. dependence of events.* Fundamental to any discussion of stochastic processes and events that constitute the process is whether two events are probabilistically independent or dependent. I will have more to say about this later.

4. *Causal relations in stochastic processes.* We can develop a good general theory of probabilistic causal relations for stochastic processes and I have tried to do so for several kinds of cases in my earlier monograph on probabilistic causality [8]. Much of that discussion can be made in terms of qualitative theory and I have, in fact, in that monograph a section on a qualitative causal algebra.

5. *Generic events.* The most important nonstandard concept for what I want to discuss here is the idea of generic events. This is not an idea that ordinarily has a technical definition within the framework of the standard probabilistic treatment of stochastic processes, but it is important, as in the case of other variable-free semantics for natural language, to have such a concept. Here I have in mind, especially for stationary processes, the way in which we talk about general properties of the process. Consider the simple example of coin-tossing. You may want to say for a given coin, "Well, for this coin heads are more probable than tails." We interpret this statement in terms of the generic event of a head and the generic event of a tail. We do not restrict ourselves to a particular time or particular trial, but make a generic claim. For such purposes, we need to be able to talk about generic events. Here is a simple way in which we can introduce such generic events. What we want to do, of course, is drop the variable for the time occurrence. I will do this in the context of discrete time, but it works just the same for continuous time. So the strict definition is as follows:

Definition: $H \succ T \text{ iff } \forall n P(H_n) > P(T_n)$.

In a slightly more technical but qualitative language, we can read this notation as simply saying "Heads are more probable than tails." Notice that I shall use here a rather simple device for creating generic events. I use the same notation as for particular events but simply drop the subscript indicating when they occur. This will not necessarily be satisfactory for all occasions, but it is important to have a simple semantic notation for a large number of familiar examples. A general remark about genericity is in order here. In the linguistic literature (e.g., Carlson and Pelletier, [2]), it is sentences, such as *Cats are smaller than dogs*, or *Birds fly*, that are generic. In the mathematics literature it is nonlinguistic mathematical objects. On the other hand, the mathematical concept of generic varies from one context to another. One of the best known definitions is that of Smale [6]. He proposes that the term be used mainly for properties of a topological space that are dense in the space. In the present case, the following probabilistic definition is more natural than the overly strong, simple one stated above. We require the definition to hold, except on sets of measure zero: the property of heads being more probable than tails is generic because it holds except on a set of trials of measure zero

$$P(H_n) > P(T_n).$$

In fact, for many ordinary examples, it will work better to weaken the requirement for being generic to hold except on a set of trials of measure less than ϵ for some positive ϵ close to zero.

6. *Concept of Occurrence.* In many discussions of stochastic processes, there is no formal introduction of the concept of occurrence but, of course, if we take an ordinary formal definition of stochastic processes as given in standard work in probability theory, we cannot define within that framework that a particular event occurred. In theoretical work, this is not important. It is important to have an explicit concept of an event having actually occurred in practical work and especially in the analysis of ordinary language. I think it may be useful, therefore, to state here some standard axioms of occurrence, which are the ones given in my earlier cited monograph [8].

Axioms of Occurrence

Axiom 1	If ΘA then $\Theta(A \cup B)$.
Axiom 2	If ΘA and ΘB then $\Theta(A \cap B)$.
Axiom 3	ΘA or $\Theta \bar{A}$.
Axiom 4	If ΘA then it is not the case $\Theta \bar{A}$.

In these axioms the notation "ΘA" is to be read, "Event A occurred." We could seek more particular axioms by indexing Θ on time t. But again, this is a more technical development that will not be followed here.

7. *Random variables.* In much philosophical discussion of probability, the central technical concept of random variable is not introduced. Here we just will say something briefly in order to have the notion in front of us, but we have in mind that we have a probability space $(\Omega, \mathcal{F}, P)$ where Ω is the set of possible outcomes, $\mathcal{F}$ is the σ-algebra of events constructed from possible outcomes in Ω, so in other words, $\mathcal{F}$ is a σ-algebra over Ω, and P is a probability measure defined on $\mathcal{F}$. Then we say that a random variable on this probability space is just a measurable function from Ω to the set Re of real numbers.

Given this apparatus, a stochastic process is just an indexed family of random variables, e.g., $\{\mathbf{X}_i: i \in N\}$, where N is the set of natural numbers.

4 Axioms for Qualitative Probability

In order to have some definite framework which will be needed in the future for various analyses of ordinary language, we make explicit the axioms we expect qualitative probability to satisfy.

1. *Axioms in terms of events.* To show some of the subtleties of the analysis, let us consider what happens in the case just of finite spaces of events. So we have a finite probability space $(\Omega, \mathcal{F}, \succeq)$, with $\mathcal{F}$ the set of all subsets of Ω, and we take as axioms:

Axiom P1	$\succeq$ is a weak ordering of $\mathcal{F}$,
Axiom P2	$\Omega \succ \emptyset$,
Axiom P3	$\Omega \succeq A \succeq \emptyset$,
Axiom P4	If $A \cap C = B \cap C = \emptyset$ then $A \cup C \succeq B \cup C$ iff $A \succeq B$.

Now the question arises, are these axioms adequate in the following sense—does there exist a strictly agreeing probability measure P such that

$P(A) \geq P(B)$ iff $A \succeq B$?

This question was first posed by de Finetti and, as is well known, the answer is negative. These axioms are not sufficient to guarantee the existence of a strictly agreeing measure, even in the case of finite probability spaces. So we turn to an extension of the notion of events.

2. *Elementary random variables.* To get an adequate formulation in very simple terms, we cannot confine ourselves just to events, but need to go on to at least a class of elementary random variables. For this purpose, we introduce first indicator functions.

$$\mathbf{A}'(w) = \begin{cases} 1 & \text{if } \omega \in A, \\ 0 & \text{otherwise,} \end{cases}$$

where **A** is an event and w is a possible outcome that is an element of **A**. **A'** is the indicator function of the event **A**. Such indicator functions by themselves just have the same structure as events because they are in one-to-one correspondence. But now, we close the indicator functions under addition, that is, under function addition, to get the class of extended indicator functions, which are the elementary random variables we will deal with. It is their qualitative expectations that we use as the basis for the new axioms and we seek a quantitative representation of expectation. This will be clear in a moment.

Our qualitative relation is now between elementary random variables. The relation $\mathbf{X} \succeq \mathbf{Y}$ means qualitatively the expectation of **X** is at least as great as that of **Y**. The axioms are as follows:

Axiom R1	*The ordering relation $\succeq$ is a weak order of the random variables,*
Axiom R2	$\mathbf{\Omega}' \succeq \mathbf{A}' \succeq \mathbf{\emptyset}'$,
Axiom R3	$\mathbf{\Omega}' \succ \mathbf{\emptyset}'$,
Axiom R4	$\mathbf{X} + \mathbf{Z} \succeq \mathbf{Y} + \mathbf{Z}$ *iff* $\mathbf{X} \succeq \mathbf{Y}$,
Axiom R5	*Archimedean Axiom, whose exact technical formulation I omit.*

From these very elementary axioms we can then prove, whether the basic probability space is finite or infinite, the following representation theorem in terms of an expectation function for the random variables [13].

(i) $E(\mathbf{X}) > E(\mathbf{Y})$ iff $\mathbf{X} \succeq \mathbf{Y}$,

(ii) $E(\mathbf{X} + \mathbf{Y}) = E(\mathbf{X}) + E(\mathbf{Y})$,

(iii) $E(\mathbf{\Omega}' \cap \mathbf{\Omega}') = E(\mathbf{\Omega}')E(\mathbf{\Omega}')$.

Furthermore, the expectation E is unique, for the requirement that equation (iii) be satisfied has as a consequence that the expectation of Ω' is 1. We then define the probability of an event just in terms of the expectation of its indicator function.

$$P(A) = E(\mathbf{A}').$$

5 Examples

Using in an informal way the apparatus introduced we now look at a number of different kinds of examples.

1. Familiar Gambler's Fallacy. *A tail following three heads is more probable than a tail following two heads.*

$$T \mid HHH \succ T \mid HH \tag{1}$$

   Notice that in this formulation generic events are assumed, and also the three heads or two heads constitute a run, with no intervening outcomes of other flips between them. This is made explicit in the standard formal notation by using successive integers as trial subscripts, such as $H_n H_{n-1} H_{n-2}$.

2. Here is an example using both kinds of inequalities introduced, one for time and one for probability. *As usual, it is more likely than not that Mary will arrive before Susan.*

$$(M > S) \succ (S \geq M) \tag{2}$$

   In this case, a little quantitative notation may produce a sense of simplification:

$$P(M > S) > \tfrac{1}{2} \tag{3}$$

   Note that (3) really says just the same thing as (2), if qualitative probability judgments are interpreted to have an equivalent corresponding quantitative formulation.

3. It is convenient to use the quantitative inequality symbol > for other quantities than time, which may be physical, psychological, economic or something else. For example, *Ordinarily Bill runs faster than Joe.*

$$B > J \succ J \geq B \tag{4}$$

   In this example, as in the previous one, we use capital letters for generic events. B stands, not for Bill, but for Bill's running, and similarly for J.

4. We can also express certainty in a natural way using generic events. *Bill always runs faster than Joe.*

$$B > J \approx \Omega \tag{5}$$

   In terms of formal inference note that (5) implies (4) on the assumption, which we implicitly make, that ≥ is a weak order of quantities and the qualitative axioms of probability stated above also hold. Here is how one elementary derivation of the result looks, where $\neg$ is event complementation or "negation":

$$\begin{aligned} B > J &\approx \Omega \\ \neg(B > J) &\approx \neg\Omega \\ J \geq B &\approx \phi \\ \Omega &\succ \phi \\ B > J &\succ J \geq B. \end{aligned}$$

5. In a purely qualitative theory of generic events it is natural to blur the important formal distinction between expectations of random variables and the probability of events. *Usually Bill weighs more than Joe.* In terms of quantitative expectation we would express this as:

$$E(B) > E(J), \tag{6}$$

where B is the generic random variable for Bill's weight at different times, and similarly for J. But in qualitative and generic terms we can just as well think of the generic event of Bill weighing more than Joe, and express the relation as

$$B > J \succ J \geq B, \tag{7}$$

as before.

6. Counterfactuals have a natural semantic interpretation when something like a probability space is given that contains all the possibilities arising in the restricted situation for which the probability space provides a model, necessarily highly simplified from many points of view. The following such example also uses a more complex tense structure, and is once again in the spirit of the Gambler's Fallacy. *If two tails had just occurred, the chance now of a head would be greater than it actually is after the head and the tail that did occur.*

$$H \mid TT \succ H \mid HT \,\&\, \theta(HT). \tag{8}$$

This is the only example thus far using the occurrence notation and implicitly thereby the axioms of occurrence given above. Others of many types are easy to construct. We also could add the explicit assertion $\neg\theta(T\,T)$, but for generic events (or generic random variables) assumed to satisfy the elementary axioms given above, this would follow from the axioms of occurrence. To feel comfortable with such inferences it is necessary to treat generic events as satisfying the time-independent axioms of elementary probability, which, of course, won't hold for generic events assumed to hold at different times, or to possess a temporal relation. The present kind of example works just because all the generic events considered are assumed to be alternative possibilities at some one generic time.

Comment on semantics and compositionality. In considering these examples I have ignored the problem of providing an explicit grammar for the fragment of English used, and, more importantly, explicitly giving the associated semantics. Given the simplicity of my examples, this is not, I think, a difficult task. The variable-free semantic functions associated with each grammatical rule of production would make evident the semantic rules of composition. To see how I would do this, I refer the reader to the relevant articles collected together in my book [11], where details for a variety of examples are given.

6 How Much Detail is Appropriate?

The examples given in no sense canvas the many different kinds that can be constructed. I have not even attempted to construct a very crude typology of cases. This is not an oversight or due to self indulgence. I am skeptical of the usefulness of trying to do so. More generally I am skeptical of the elaborate algebraic structures of events that have been constructed in the linguistic literature—for good examples see Bach [1] and Link [4]. In spite of the many interesting constructions, linguistic examples and semantic puzzles presented in the two papers referred to, and others like them, I have not followed their lead, but have moved in a different direction. I will try to explain why.

I have started from the attitude prevalent in probability theory and its applications. If I want to study, say, the distribution of height and weight in a given population of children at a given time, and later how this distribution changes over time, I do not begin by constructing some elaborate set-theoretically complicated probability space. I take a much more scientifically opportunistic approach by just introducing two random variables, one for height and one for weight. If for some reason I want a more detailed analysis I can introduce a random variable for each member of the population, or each individual under study. The important point is that no complicated sample space is introduced, and the metaphysics of what is assumed is left completely unstated.

This use of random variables is widespread in probability theory that has any mathematical flavor, and it is so for good reason. In many situations the explicit characterization of what is intended to be the underlying probability space is complicated and the complications are not interesting. All that is required for applications of probability theory in proper form is knowledge that there exists a sample space on which the random variables in question have a joint distribution.There is a fundamental theorem of Kolmogorov's that states that if all finite families of random variables for a given stochastic process have such a character, then there exists a probability space for the entire process.

But notice how relaxed the ontology is. There is no commitment to the details of the sample space, what properties or "objects" are included. All that is required is just that there be such a probability space. Some can make it rich and some can make it austere. But the exact metaphysics assumed is of little interest for any serious application of the probability and accompanying statistical analysis.

So it is along these lines that I part company with the kind of analysis given by Bach and by Link. They introduce an elaborate ontology, what might be called an attempt to give a metaphysics underlying the ordinary language of events. My point is not that their metaphysics is wrong, but the more general point that it is a mistake to try to give such an elaborate metaphysics.

I want to be clear about my criticisms of the work of Bach and Link. Many interesting problems are raised and the puzzles presented are often genuine ones. They present problems of analysis for anyone interested in natural language.

A typical example is what is called the minimal parts problem, discussed in various publications by both Bach and Link. This can be regarded as a problem that goes all the way back to the conflict between Aristotle and the early atomists. And if we want to make everything explicit, we will have difficulty with it. My point is that almost all natural language does not make a commitment either way about this. We do not need a commitment on what are the minimal parts in order to understand and judge an utterance as correct, as an intelligible question or as a command that can be satisfied. We operate with a much more minimal set of metaphysical assumptions in dealing with individual sentences used in a given context on a given occasion. Any metaphysics that is needed is very fragmentary and partial, computed on the spot to deal with the expressions in the utterance in question.

It is only in what I call the framework of classroom language, or what I sometimes call, even more specifically, logic classroom language, that we can begin to attempt to formulate language in such a way that we can talk about the models of it with any precision. Only then can we examine what kind of at least limited ontological commitments are made for the language as a whole by the models we consider. In ordinary talk, we talk about one thing and then quickly about another. The sentences of discourse do not have to be connected in any rigorous way, concerning truth or falsity. The subject matter changes rapidly and drastically. We have methods when we need them, for judging truth or falsity of particular utterances and some general minimal concepts are brought along from one occasion to another, but what we do not have, and do not seem to need, is a general metaphysics or a general model underlying our use of natural language. In particular, in reference to my thesis here, we do not need it for events or the stochastic processes of which events are a part. It is exactly to dodge these kinds of commitments and to go along a very smooth mathematical way, that the apparatus of random variables has been created.

Finally, let me end on a point that comes back to something I mentioned just above. In terms of what I said myself just a few paragraphs earlier, a response might be "Well, at least you have to think about the probability space to determine whether you have a proper framework for the random variables you introduce." In theory this is a problem, but in practice it never arises in ordinary applications. In fact, almost the only place I know of that has any extensive questioning of the existence of a joint distribution, that is the mere existence, of the joint distribution of random variables in question, and therefore the existence of a proper underlying probability space, is in the extensive literature on hidden variables in quantum mechanics. Even there the problem is often not posed this way, but only by probability purists such as myself. In applications elsewhere, in science and in practice, such problems do not arise. A formal underlying probability space remains back of the scene, unspecified, not even known in any particularity, only by Kolmogorov's theorem recognized to exist.

We can in fact extend this point, and this is, indeed, my last remark, to what I've had to say about stationary processes as families of random variables. Ordinary statements about expectations, about present, past or future events, do not require specification of a full stochastic process. So not only at the level of probability spaces, but also at the more abstract level of random variables, we still do not need for discussion of most ordinary language sentences about events and processes an exact specification. That is exactly why I have introduced the concept of generic event, and we could, if desired, introduce a concept of generic process. We do not need to go much deeper in the interpretation of most qualitative statements about events and processes. When we do, the apparatus is ready at hand, and in every case, when it comes to the introduction of entities, a minimalist attitude of austerity should prevail. It goes without saying, but I will say it any way. Such austerity is tailor-made for variable-free semantics.

7 Epilogue: Comments on the Philosophical Literature

There is a large philosophical literature on events. The topics range from the place of events in the theory of action to their role in the mainly informal semantic analysis of the many kinds of ordinary-language sentences that are about events. From the perspective from which I write, the most striking thing about this literature is its almost total neglect of the formal concept of event as used in probability theory and in the great variety of scientific applications of probability.

A systematic survey is out of the question here, but I can, I think, narrow the gap between this scientific viewpoint and almost all the philosophical views by considering a few much discussed examples.

Fine-grained vs. coarse-grained descriptions of events. Do *Mary sings* and *Mary sings loudly* describe one or two events? Under the fine-grained view the answer is two; one under the coarse-grained account. Compare the following two familiar coin-flipping events in three tosses of a coin.

E_1 = the event of at least two heads.
E_2 = the event of exactly two heads.

In this case, nearly everyone would agree that $E_1 \neq E_2$. In fact, the two events have different probabilities and are realized by different sets of possible outcomes:

E_1 = $\{HHH, HHT, HTH, THH\}$
E_2 = $\{HHT, HTH, THH\}$

The reply might be that, well, yes, this is coin flipping, gambling and betting, something very different from talk about Mary singing. My claim is that this attempt to separate the cases in this way is a mistake. Ordinary talk is used on many different kinds of occasions to express opinions about probable and unprobable outcomes, predictions about future events, or even quantitative expectations, which are best described formally in terms of random variables rather than events. In this framework of language use, it is easy to imagine bets about Mary tomorrow.

E_3 = Mary sings at the concert.
E_4 = Mary sings loudly at the concert.
E_5 = Mary sings too softly at the concert.

We could bet on any three of the events. In the usual Boolean notation

$$E_4 \subset E_3,\ E_5 \subset E_3,\ E_4 \cap E_5 = \emptyset.$$

Now, the response might be that this looks like the coin-flipping example, but it really is quite different, for the possible outcomes of the sample space are really never described explicitly for singing performance, but for coin flipping are part of every elementary presentation of probability theory. To this possible comment, I have two responses. First, as I have emphasized in some detail earlier, it is just a feature of the familiar random-variable formulations of more advanced probability, pure or applied, that an underlying sample space and its clearly defined set of elements, the possible outcomes, are not actually defined and used. As the stochastic processes become ever more complicated, the desire to describe an underlying sample space and its algebra of events rapidly wanes. But, on the other hand, if two critics of Mary's singing wanted to place bets on her singing at the concert in question, they could put together a set of possible outcomes, no more abstract or artificial.

Of course, not only in ordinary talk, but in scientific work as well, there is a great range in the definiteness and sharpness with which problems are formulated or solved. The features of singing are less easily defined than those of coin flipping, at least than the features in which we are interested.

In the general conceptual use of talk about expectations and predictions there is, in my mind, no sharp or important distinction between ordinary and scientific talk. Possible events, and their explicit set-theoretical construction, work about as well in the one case as the other.

Logical form of sentences about actions or events. Philosophers are much addicted to engaging in the game of giving the logical form of various kinds of sentences. The presumed aim of finding such forms is to explicate the meaning of the sentences considered by interpreting them in terms of a language such as first-order logic that has, it is assumed, a well-defined and well-developed semantic theory. I have expressed my skepticism about this general enterprise many years ago [9]. Here, I concentrate on more particular issues. From Davidson [3] to Parsons [5] the inevitable move is to find security in the predicates and quantifiers of first-order logic. To nonbelievers it looks like a new form of rather empty scholasticism. The sense of security comes from the clear and simple rules of composition of the semantics of first-order logic. What is totally missing, and what seems essential for a rich theory of meaning, is the analysis of lexical meaning, which is facilitated hardly at all by translation into first-order notation. Consider again, the sentences about Mary's singing. Parsons [5: Ch. 7] gives a wonderfully detailed analysis of the logical form of such sentences. What is missing, and apparently unmissed in this tradition, is a detailed and nuanced account of the many familiar physical, psychological and social features of singing, most reasonably thought of as part of the lexical meaning of the various verbs, adverbs, etc. used in descriptions.

A criticism of logical form from another direction is its obvious psychological artificiality. That our mental analysis of meaning much resembles current accounts of logical form seems extremely doubtful.

Issues about concrete events. Folklore physics and psychology strongly support the idea of irreducibly concrete events. An angry driver tells the policeman, "His car hit mine in the middle of the right front door. It was his fault, and nobody else's." The two drivers and the policeman both agree a particular accident, as we might say, occurred, even if there is no agreement on who is at fault.

But what is this unique concrete event that was supposed to occur? From the earlier discussion of the nonunique sample spaces supporting random variables, it should be apparent what my response to the question is. There is no such concrete event given to us by nature. We cannot literally and accurately say of some description we offer, "This is *exactly* what happened in the accident." It is a myth of the law, useful in its way perhaps, but still a myth, that witnesses can really hold to their oath to tell the truth, the whole truth and nothing but the truth about an accident, a person, or anything else.

Our descriptions of what occurred are, in principle, always partial and incomplete. We describe events alright, but never the full-blown concrete happening, which is, to give the discussion a Kantian flavor, a *ding an sich* we cannot fully comprehend.

A familiar philosophical move at this point in the analysis is to introduce space-time point events, which are much used in the mathematical analysis of both classical and relativistic physics. For example, a basic axiom that holds for both classical and special-relativity space-time is: *The set of space-time points, together with the relation of betweenness, is a four-dimensional affine space* [12].

Such space-time points are a very useful mathematical idealization but they scarcely qualify as full-bodied concrete events. Moreover, there are new problems of a more troubling conceptual kind if we try to conceptualize concrete events as what happens just at a given space-time point. I have in mind the peculiar behavior of the quantum vacuum as we try to isolate an ever smaller region of space-time, with a single point as the asymptote.

For the purposes of ordinary experience or particle physics, the concept of concrete events is often a useful fiction, but not something to be used as bedrock on which to build other concepts. Fortunately variable-free semantics works well on the shifting sands of experience without needing or using the nebulous notion of a rock-hard concrete foundation.

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